Static and User-Extensible Proof Checking

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Proof assistants are becoming popular in our community

— CompCert [Leroy et al.]

— seL4 [Klein et al.]

— Four-color theorem [Gonthier et al.]

… but they're still hard to use

— 1 to 1.5 weeks per paper proof page

— 4 pages of formal proof per 1 page of paper proof

[Asperti and Coen '10]

Formal proofs

—communicated to a **fixed** proof checker

—must spell out all details

—use domain-specific lemmas

≠

Informal proofs

—communicated to a person

—rely on **domain-specific intuition**

—use “obviously”

Formal proofs

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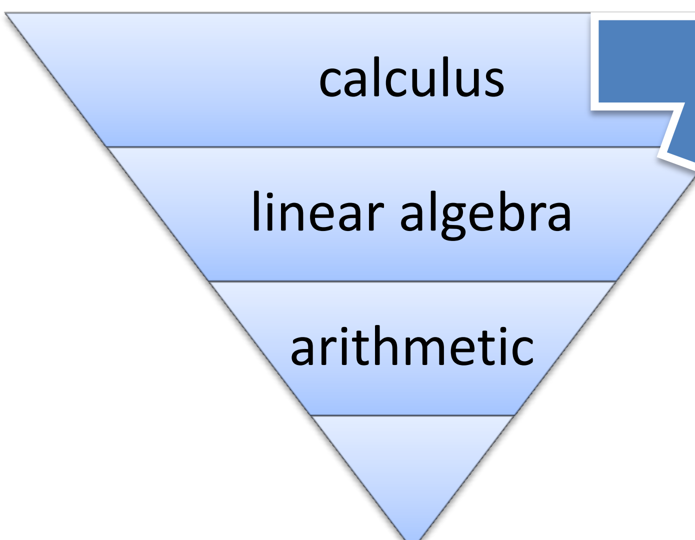

≠

Informal proofs

–communicated to a person

—rely on **domain-specific intuition**

—use “obviously”

We need tactics to omit details

— procedures that produce proofs
— domain-specific tactics good in large developments
— but difficult to write!

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—procedures that pro

—domain-specific tact
developments

—but difficult to write!

- untyped
- proofs within tactics can be wrong!

Proof assistants are hard to use because

1. cannot extend proof checker → lots of details

2. no checking for tactics → lots of potential errors

These are architectural issues

Our contribution:
A new architecture for proof assistants

1. cannot extend proof checker → lots of details
2. no checking for tactics → lots of potential errors

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1. ~~cannot extend proof checker → lots of details~~

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full programming model
soundness guaranteed

contribution:
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2. extensible checking for tactics → lots of errors avoided

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static checking of contained proofs

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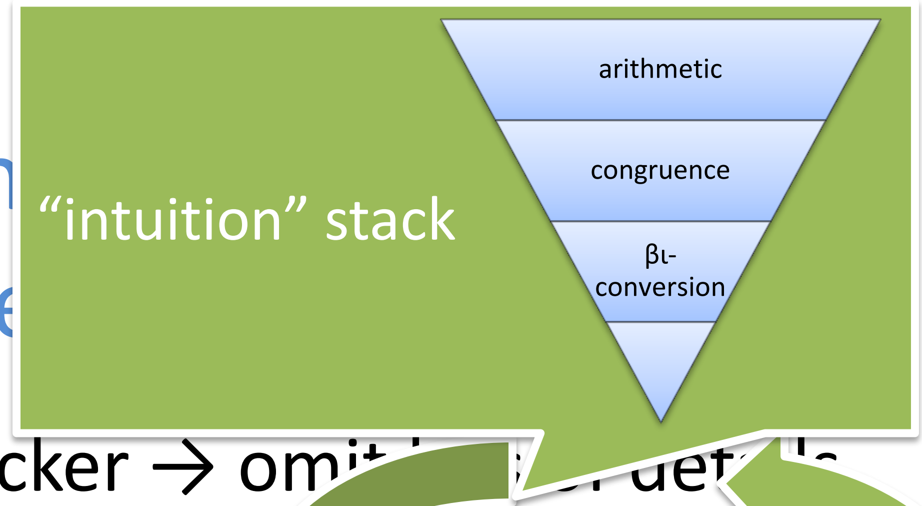
Our con…
A new architecture…



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More specifically:

- **a new language design**
- **a new implementation**
- **and a new metatheory**

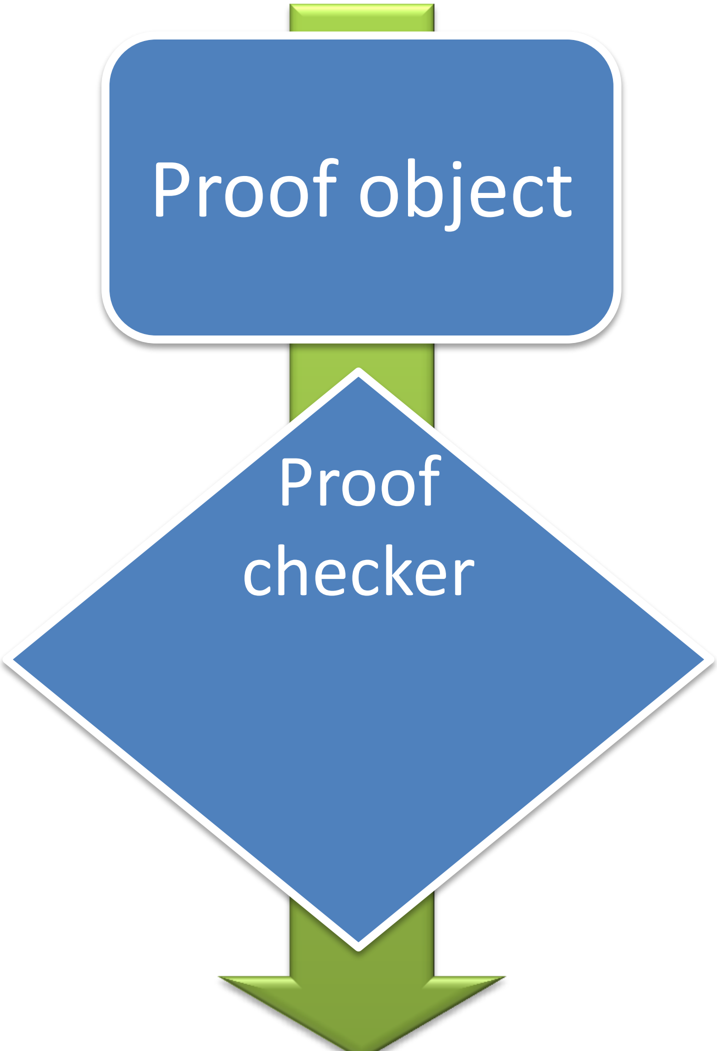
based on VeriML [ICFP'10]

Architecture of proof assistants

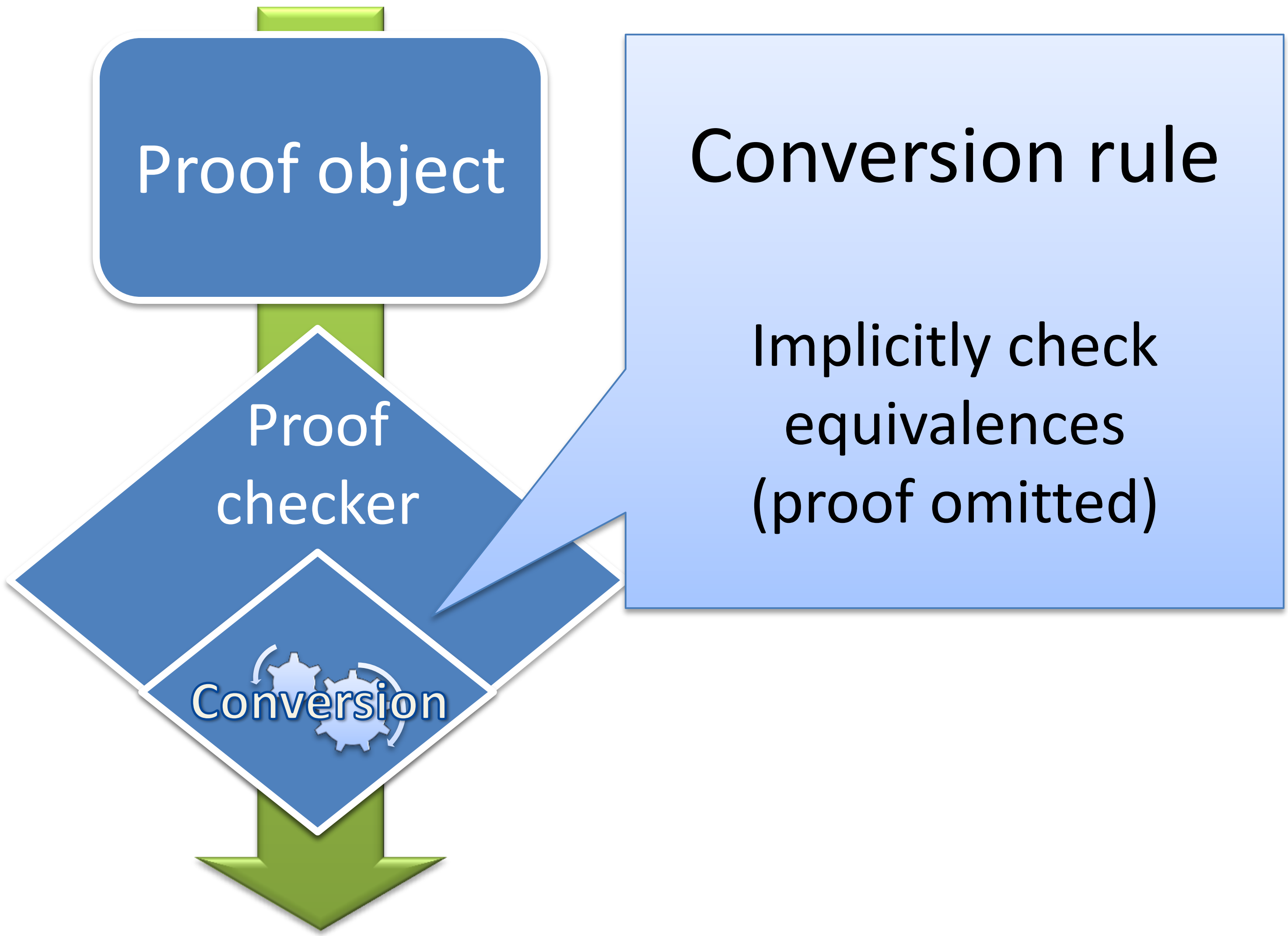
Architecture of proof assistants: main notions

Proof object	Derivation in a logic
Proof checker	Checks proof objects
Tactic	Function producing proof objects
Proof script	Combination of tactics; program producing a proof object

Architecture of proof assistants: Checking proof objects



Architecture of proof assistants: Checking proof objects



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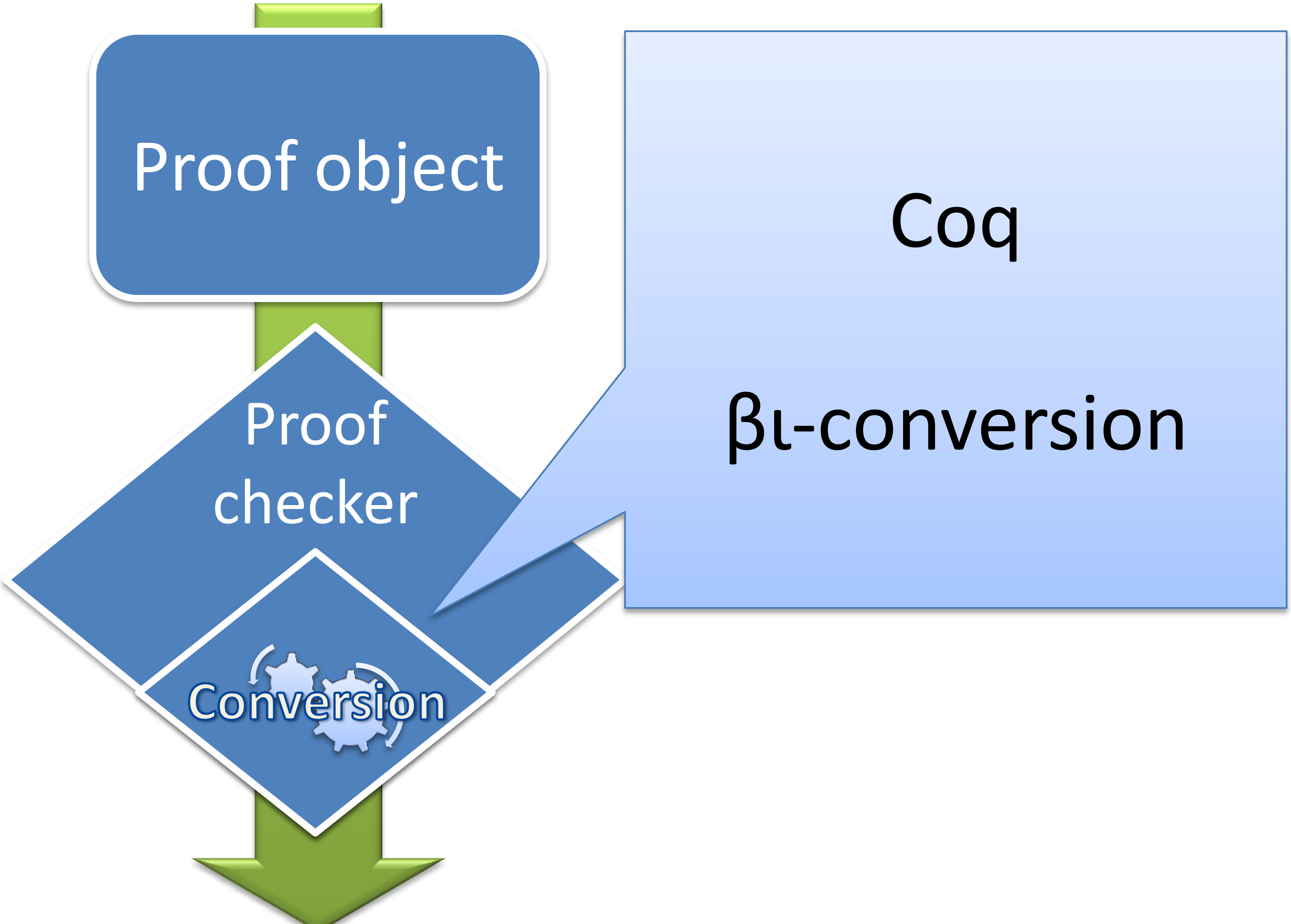

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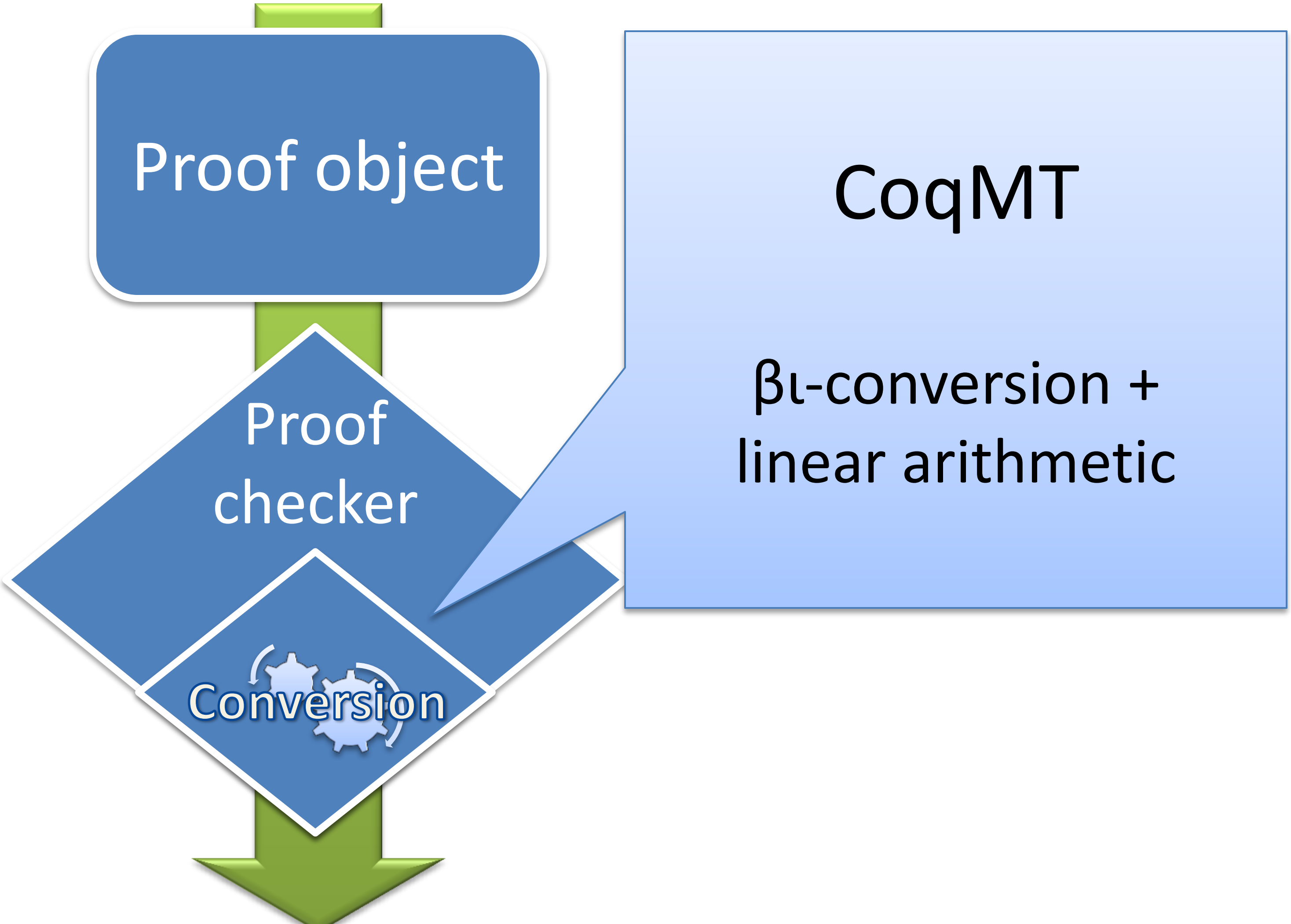

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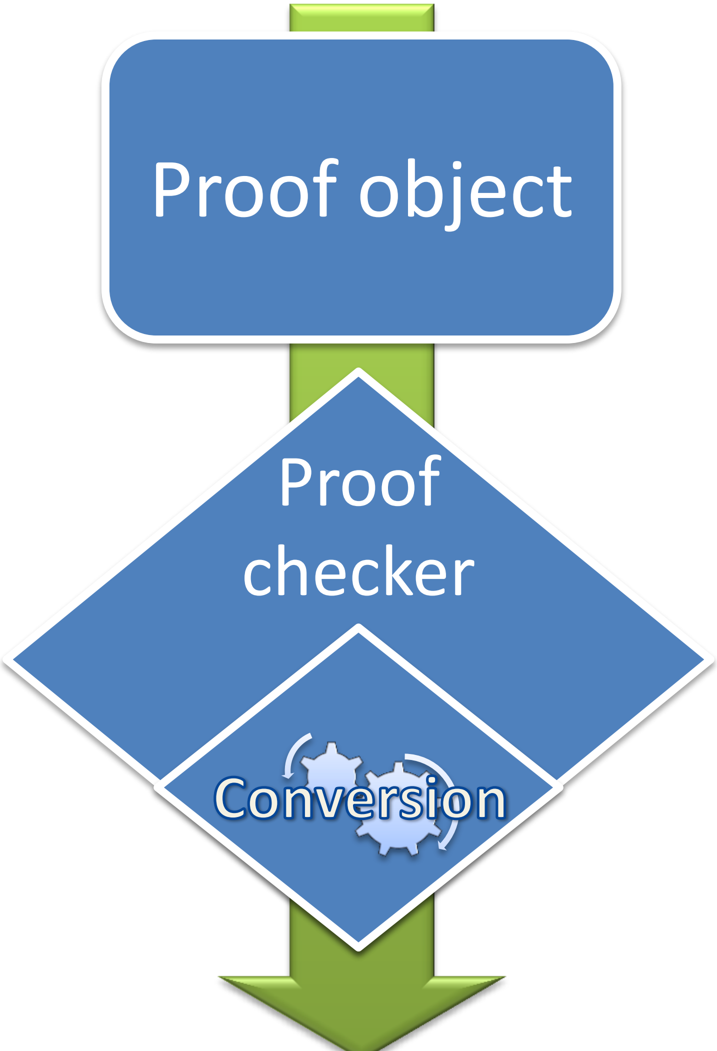
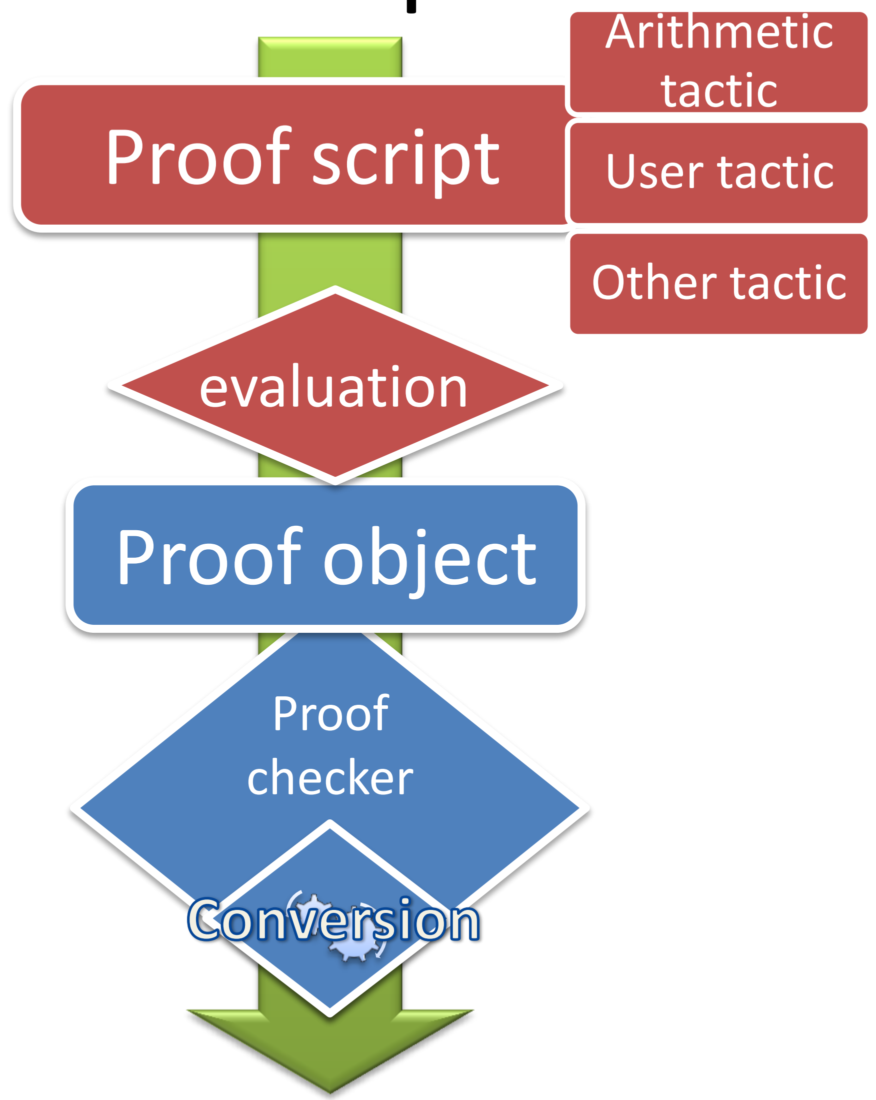


- rich static information
- (de)composable
- checking not extensible

Architecture of proof assistants: Validating proof scripts

- extensible through tactics
- rich programming model
- no static information
- not (de)composable
- hidden proof state



Architecture of proof assistants: Validating proof scripts

- extensible through tactics
- rich programming model
- no static information
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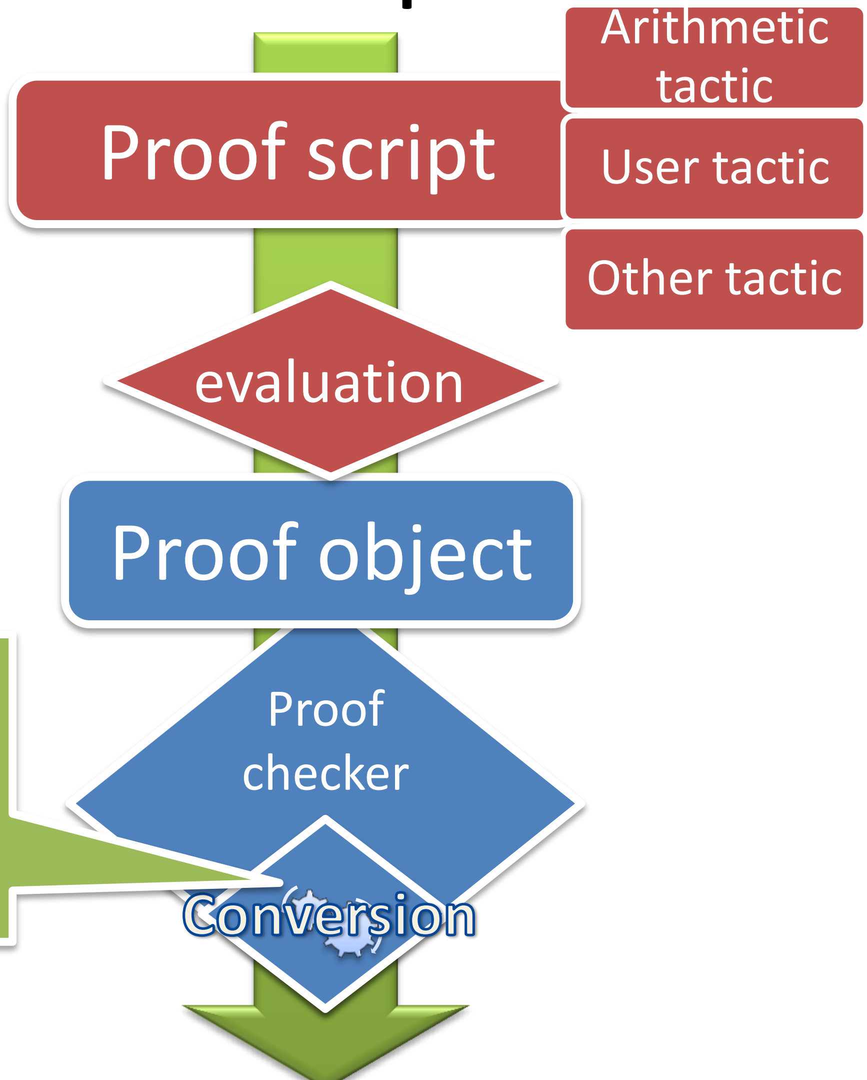
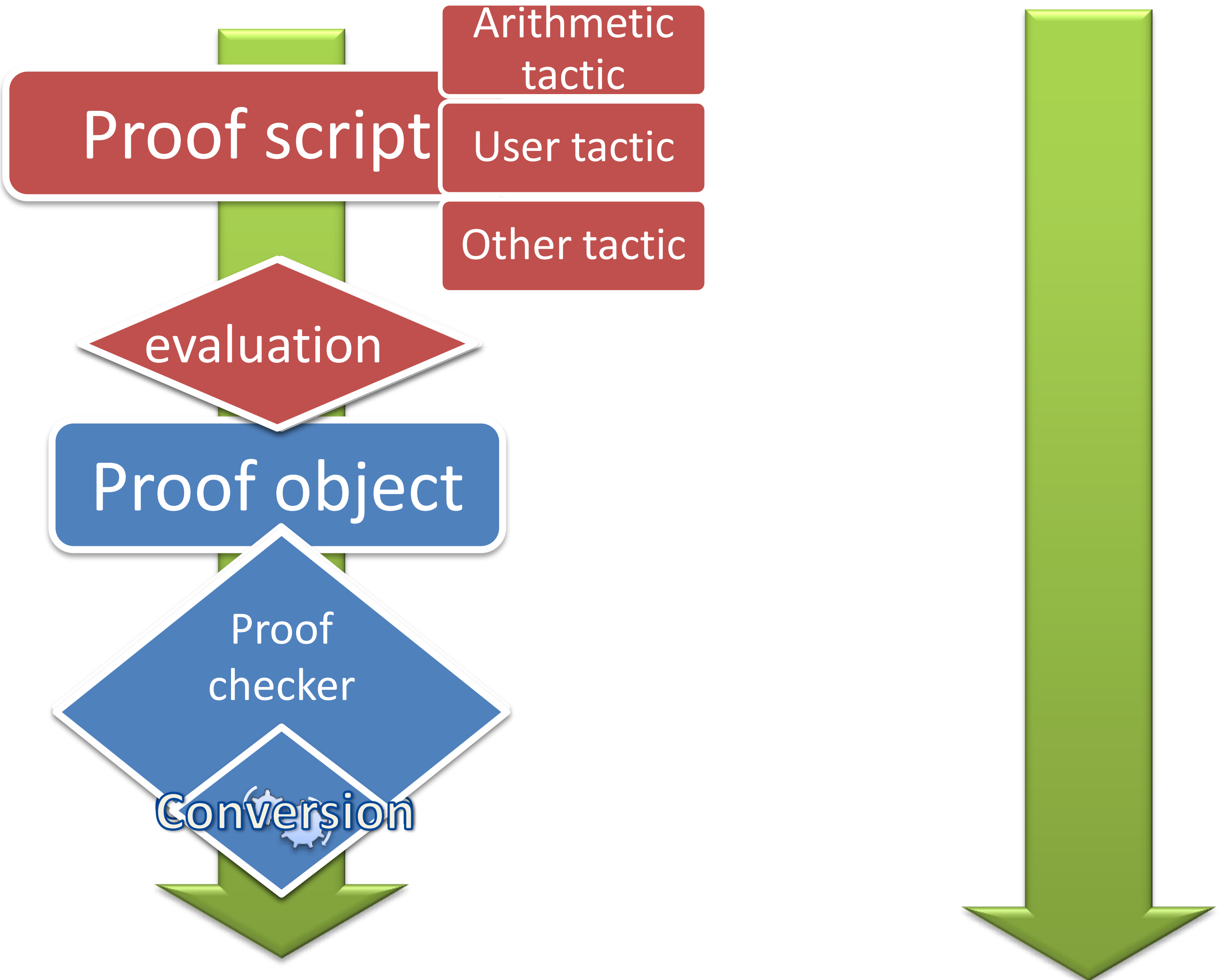
use conversion for more robust scripts



Moving to typed proof scripts



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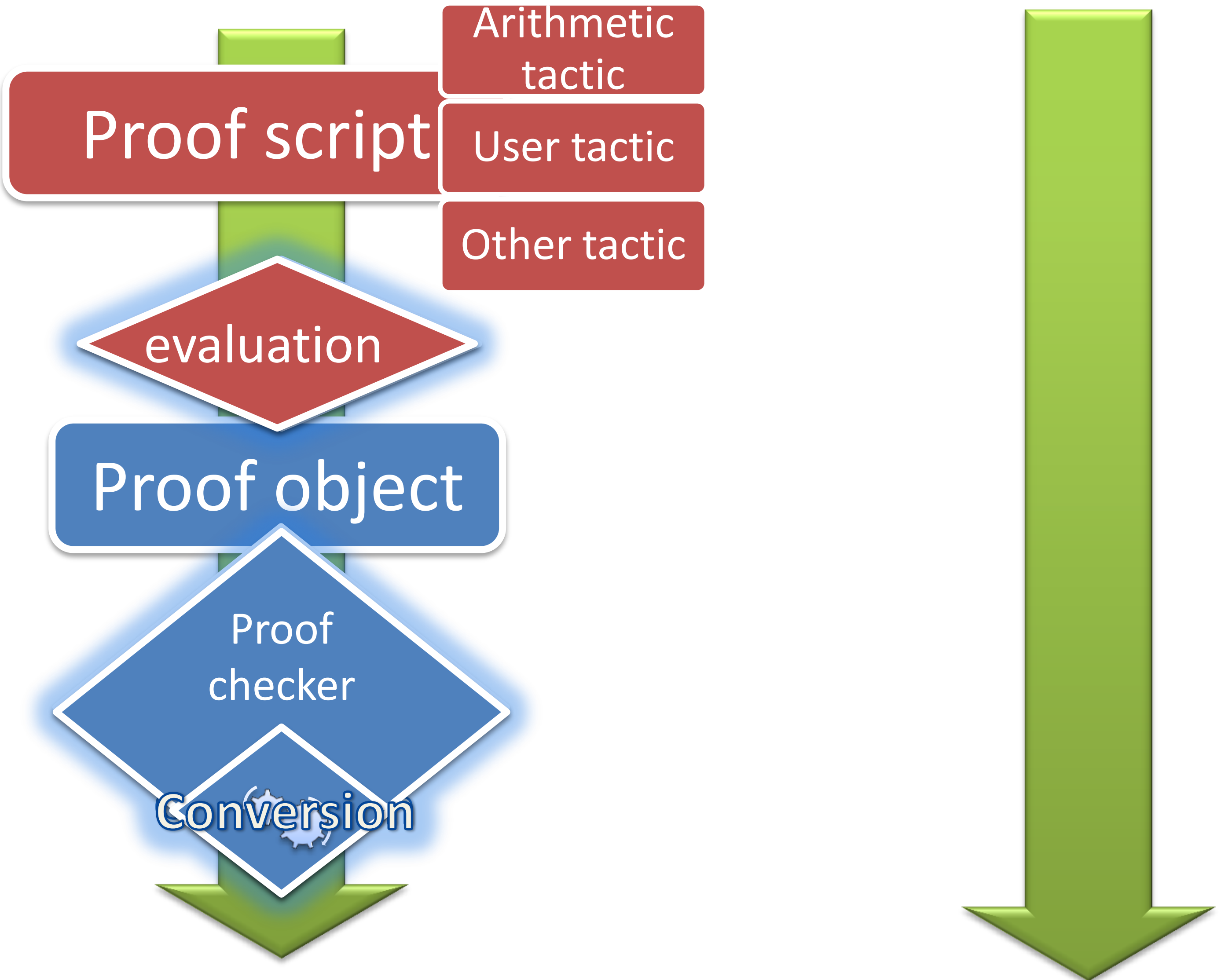

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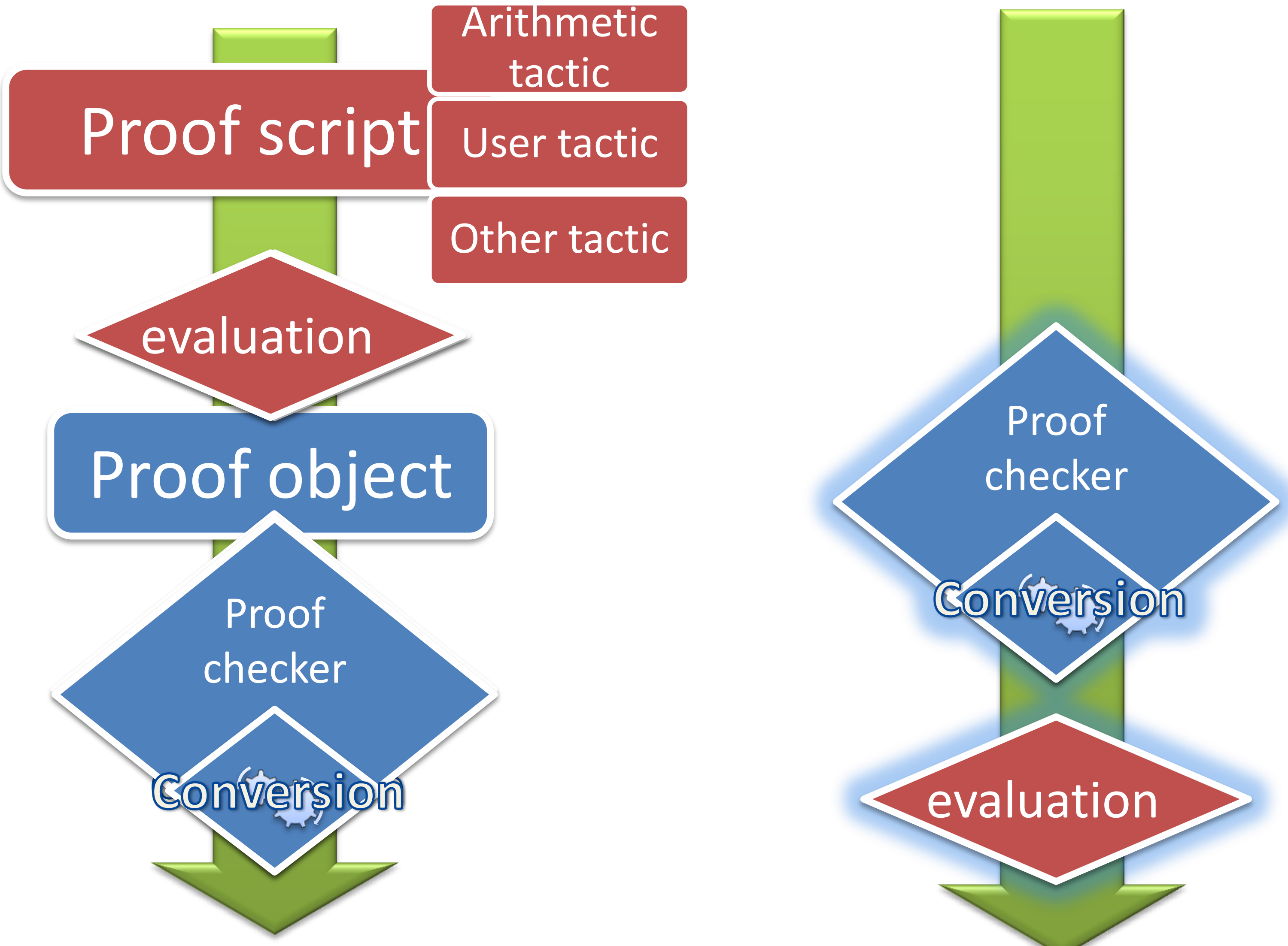

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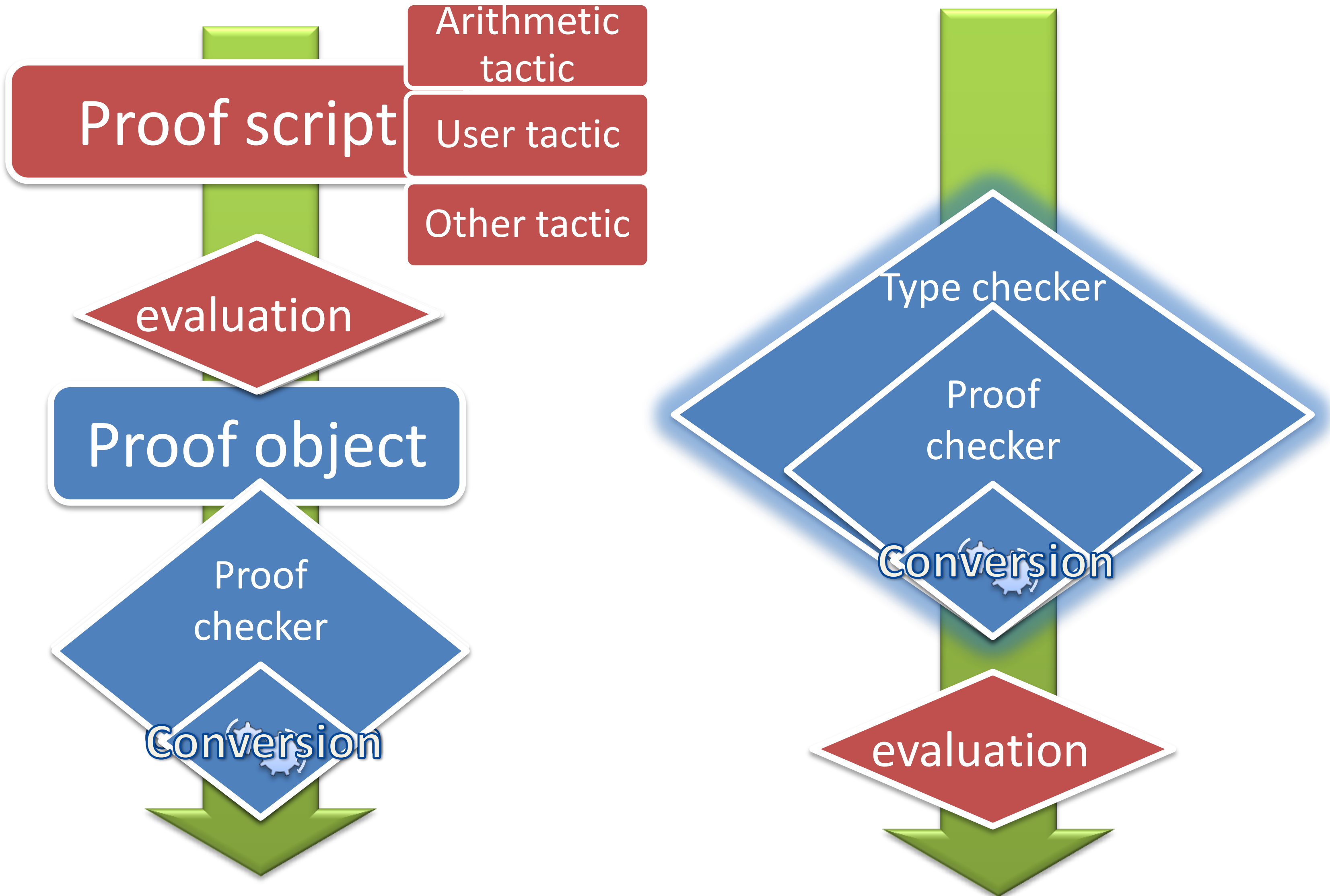

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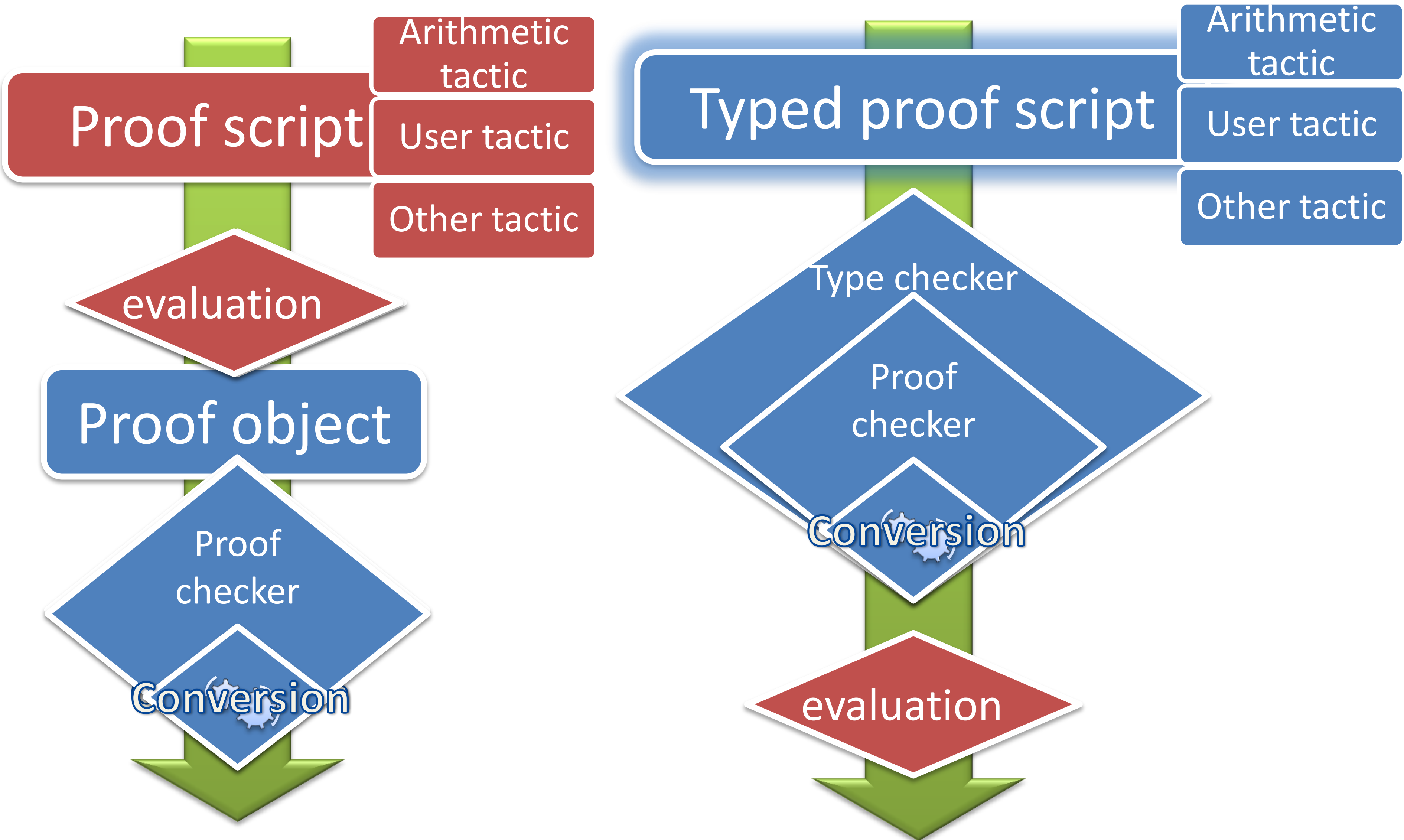
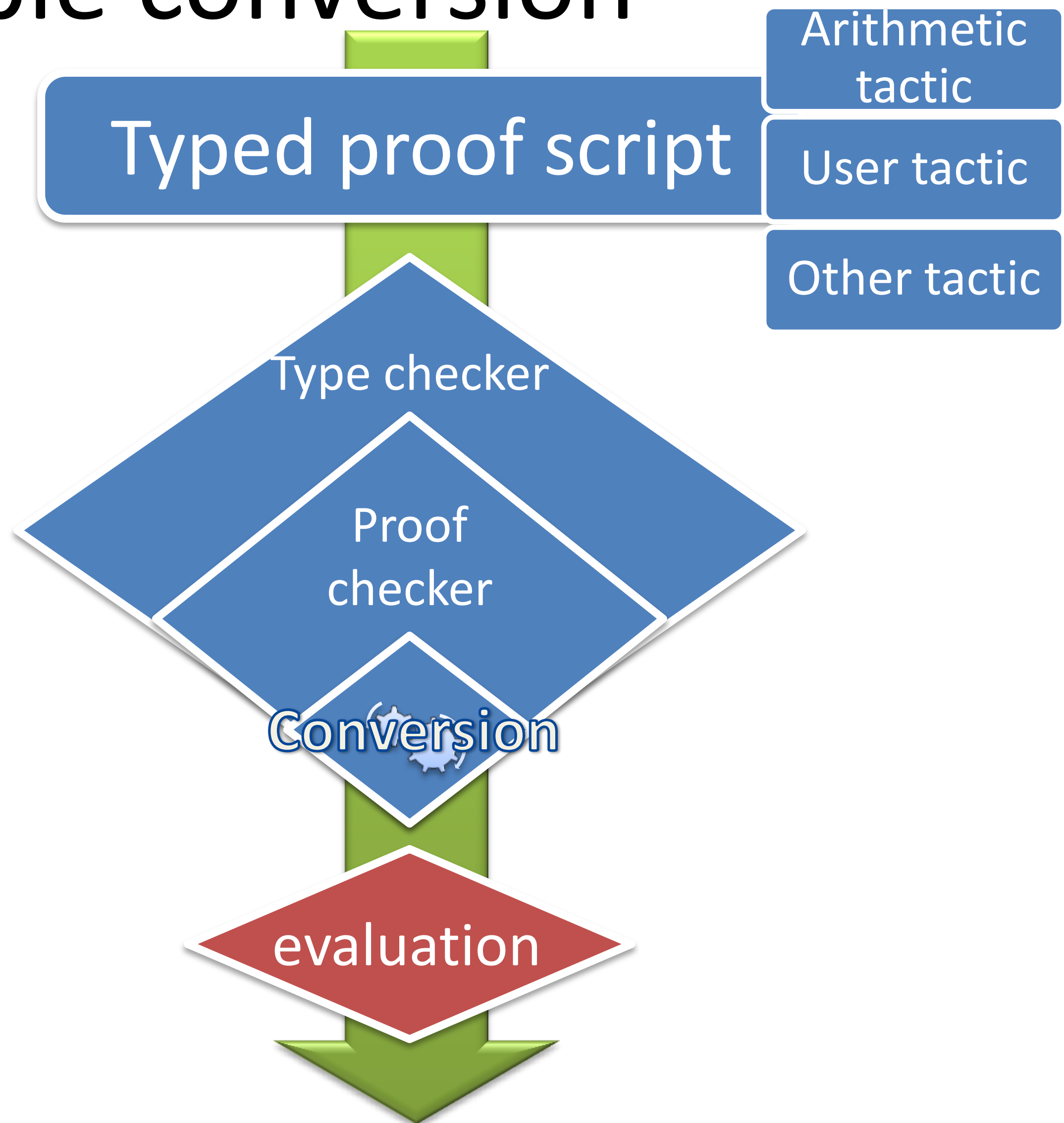
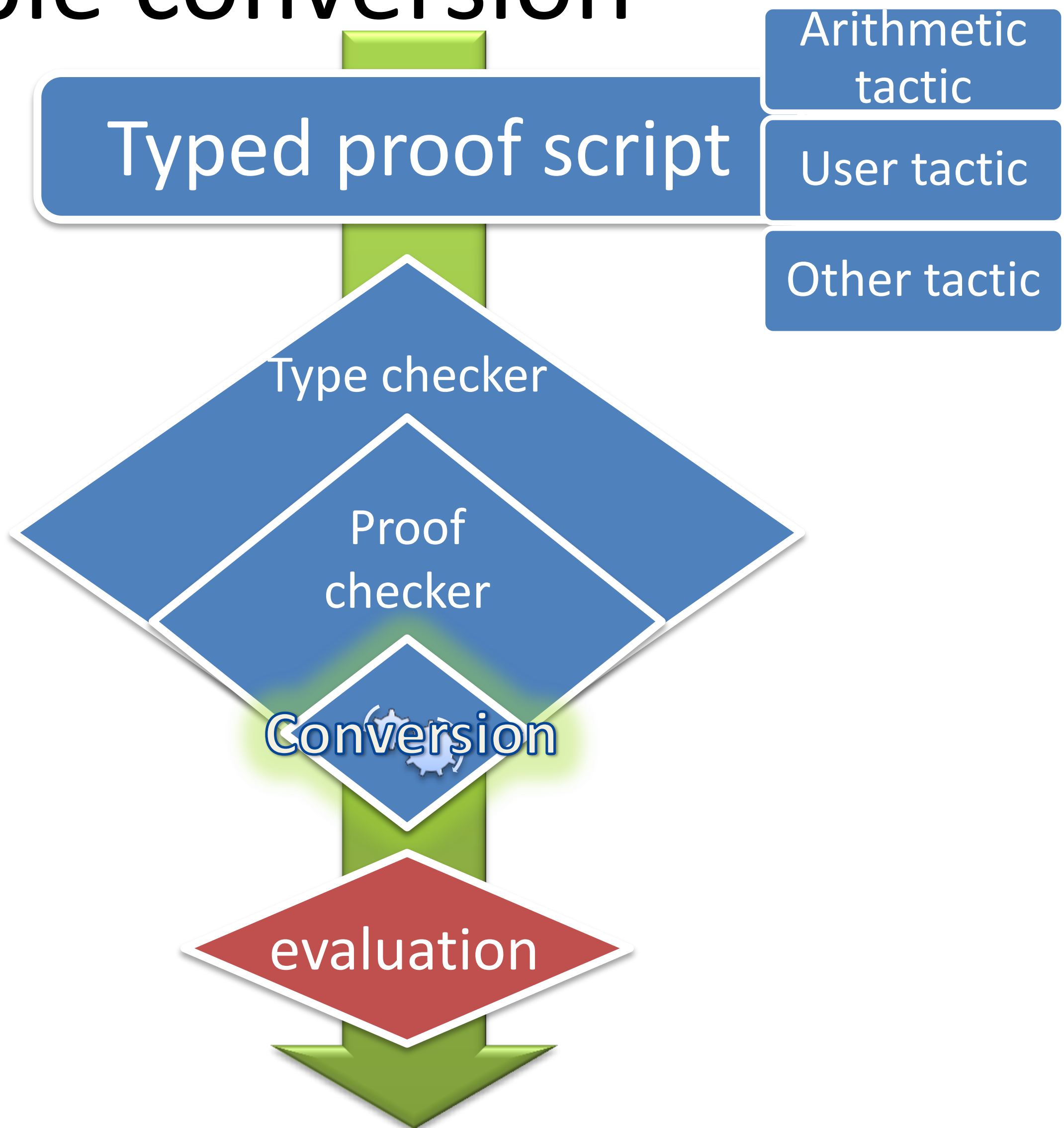


Moving to typed proof scripts + extensible conversion



Moving to typed proof scripts + extensible conversion

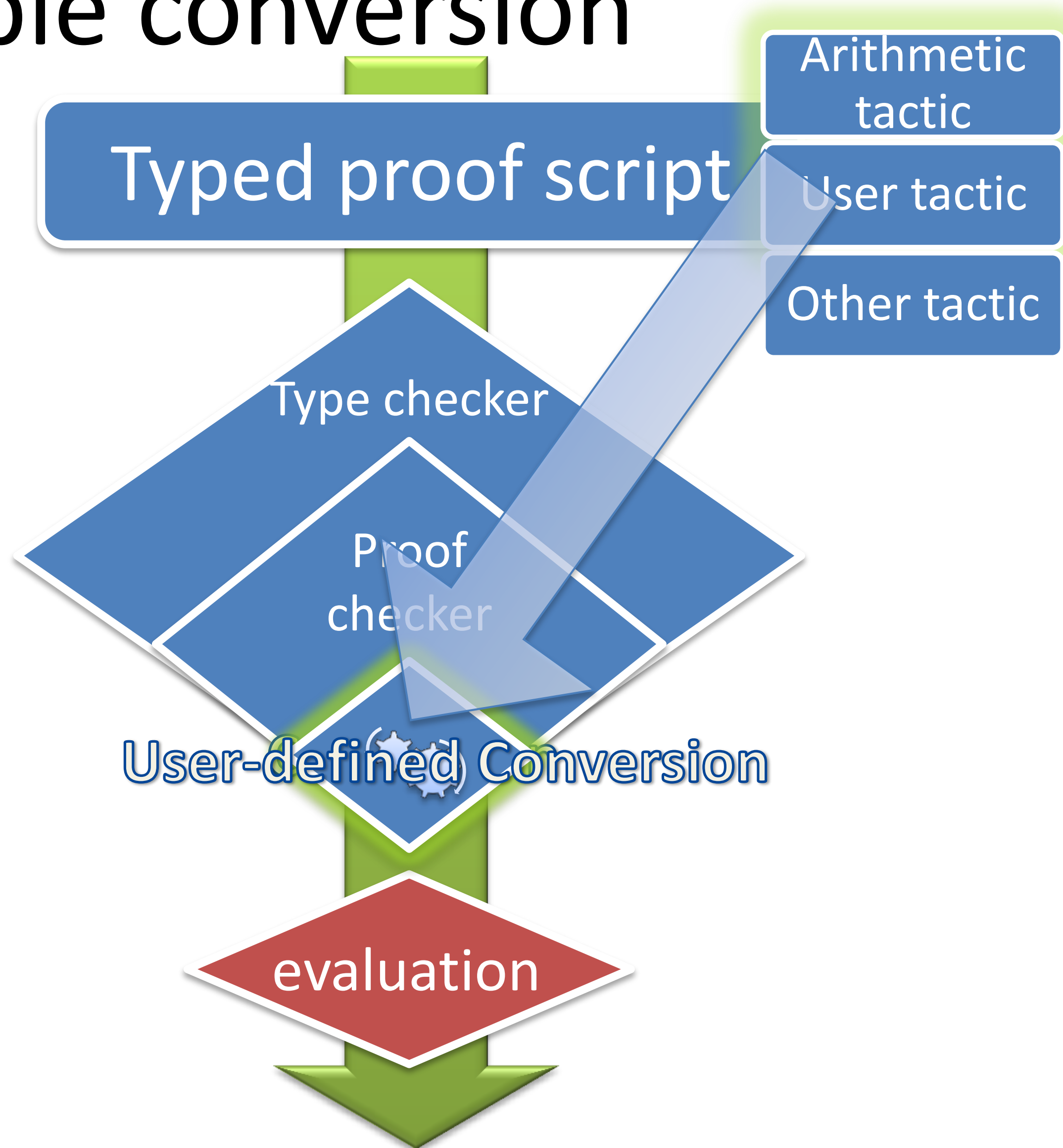
Key insight:
conversion is just a hardcoded trusted tactic



Moving to typed proof scripts + extensible conversion

Key insight:
conversion is just a hardcoded trusted tactic

but we can trust other tactics too if they have the right type

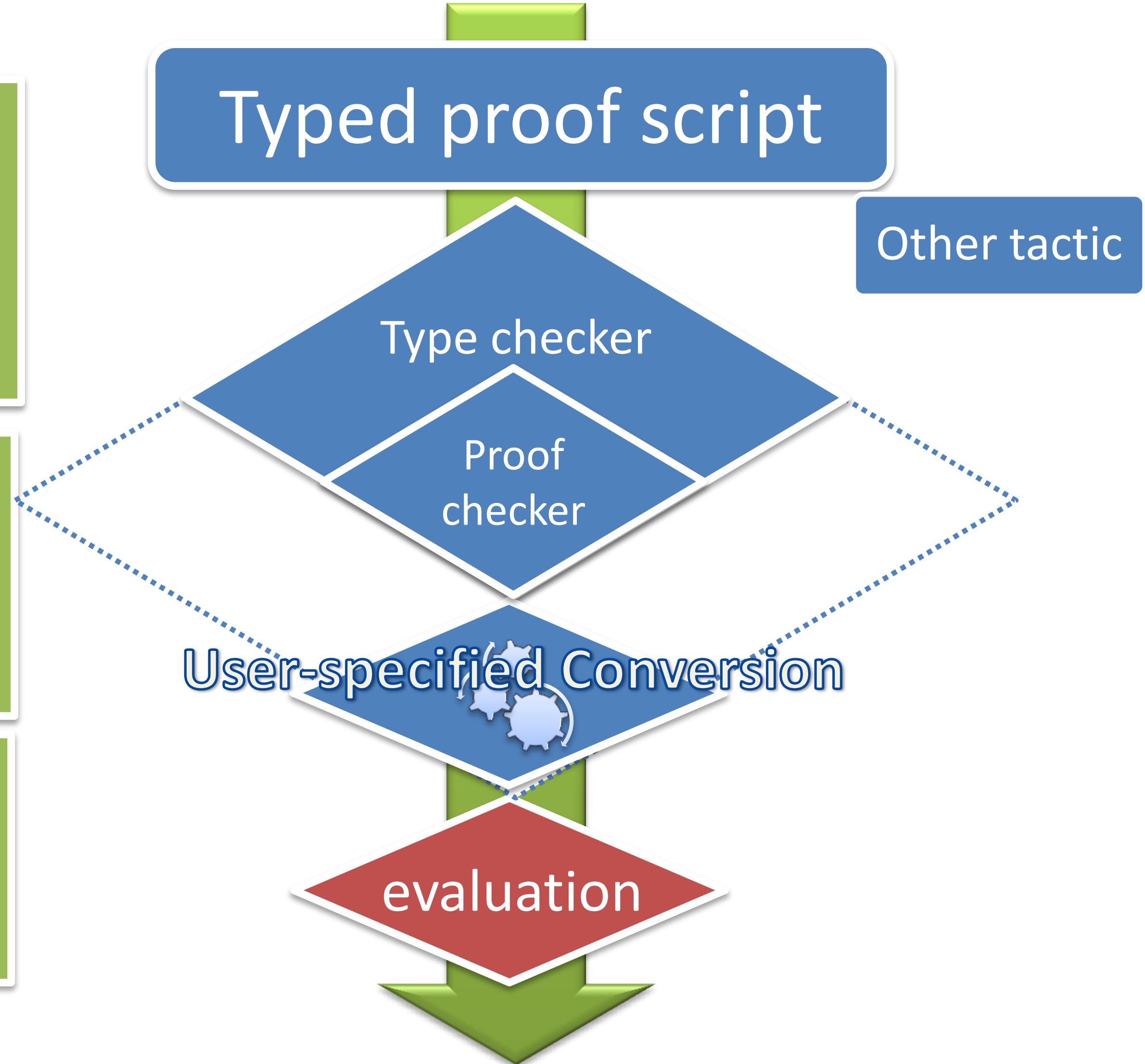


Moving to typed proof scripts + extensible conversion

Key insight:
conversion is just a hardcoded trusted tactic

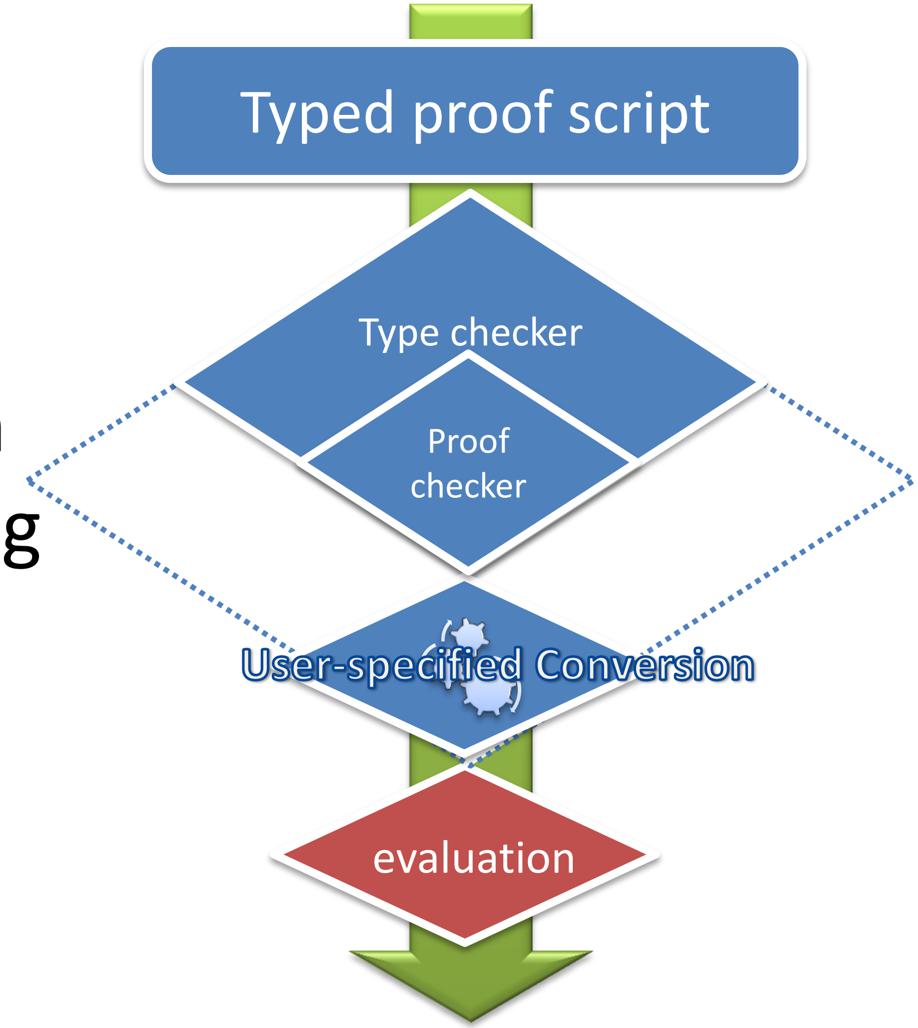
but we can trust other tactics too if they have the right type

none of them needs to be hardcoded!



Typed proof scripts + extensible conversion

- rich static information
- user chooses conversion
- extensible static checking
- smaller proof checker
- can generate proof objects



Type checking tactics: an example

$$\mathsf{conversionCheck} \quad : \quad (P : \mathsf{Prop}) \rightarrow (P' : \mathsf{Prop}) \rightarrow (\mathsf{proof}{:}P = P')$$

- check propositions for equivalence
- return a proof if they are
- raise an exception otherwise

Metatheory result 1. **Type safety**

If evaluation succeeds,

the returned proof object is valid

T… an example

$$\text{conversionCheck} \quad : \quad (P : \text{Prop}) \to (P' : \text{Prop}) \to (\text{proof:} P = P')$$

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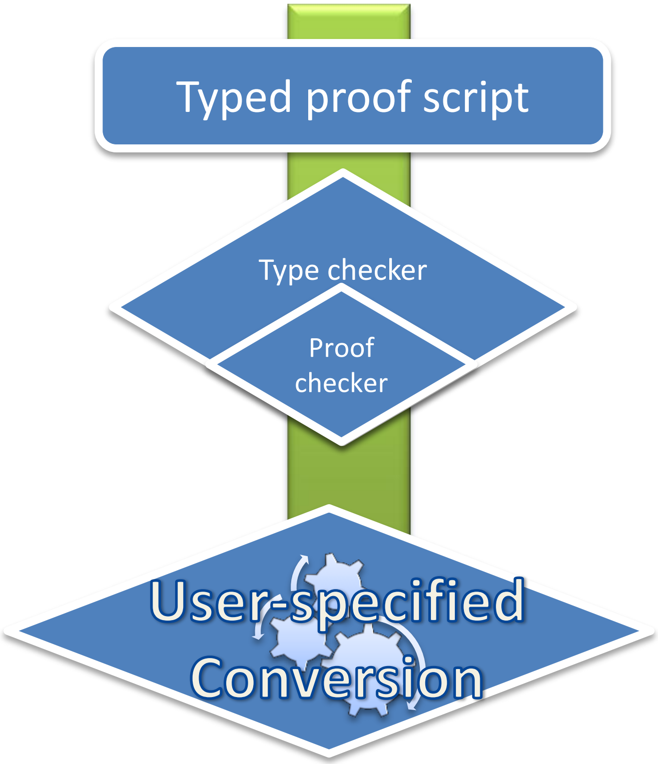
$$\mathsf{conversionCheck} \quad : \quad (P : \mathsf{Prop}) \to (P' : \mathsf{Prop}) \to (\mathsf{proof}{:}P = P')$$

Metatheory result 2. **Proof erasure**
If evaluation succeeds, a valid proof object exists even if it's not generated

- chec...
- retu...
- raise an exception otherwise

Two modes of evaluation



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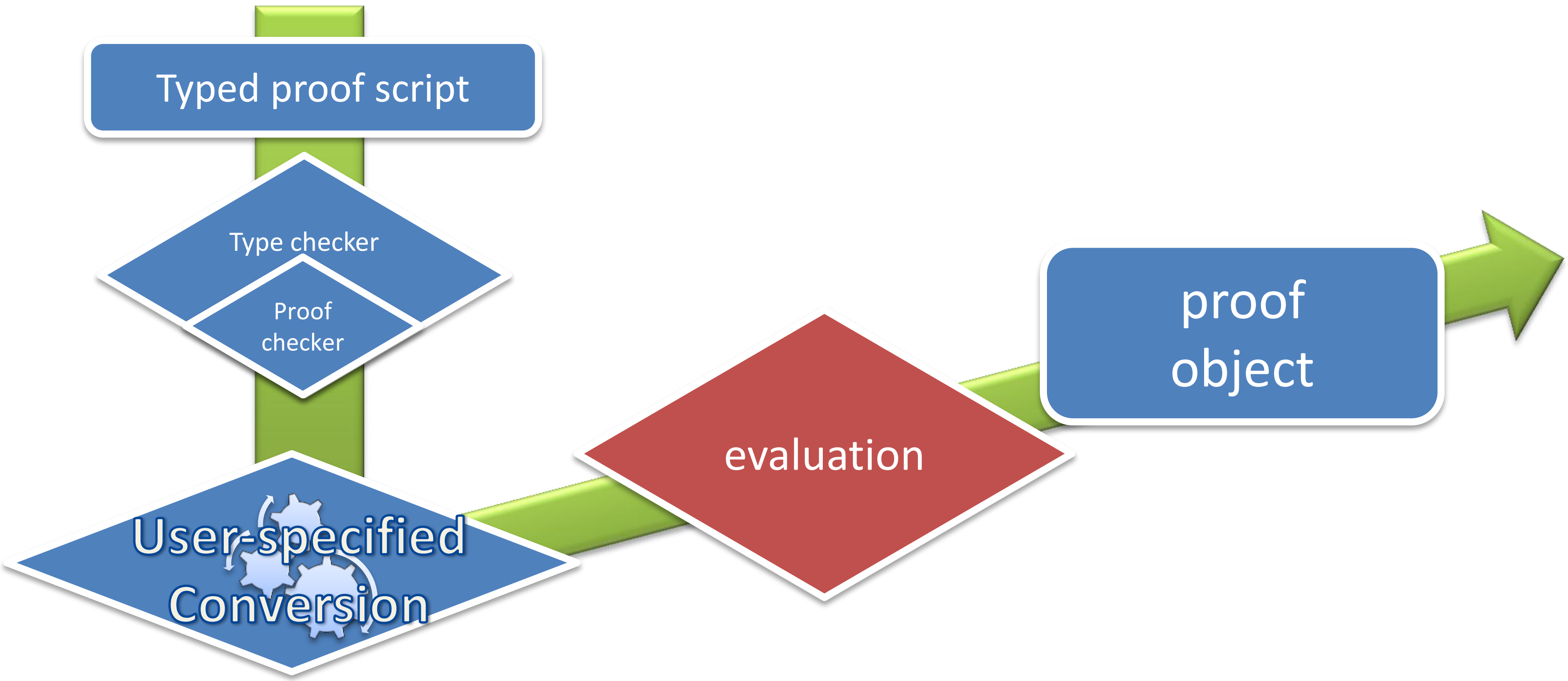

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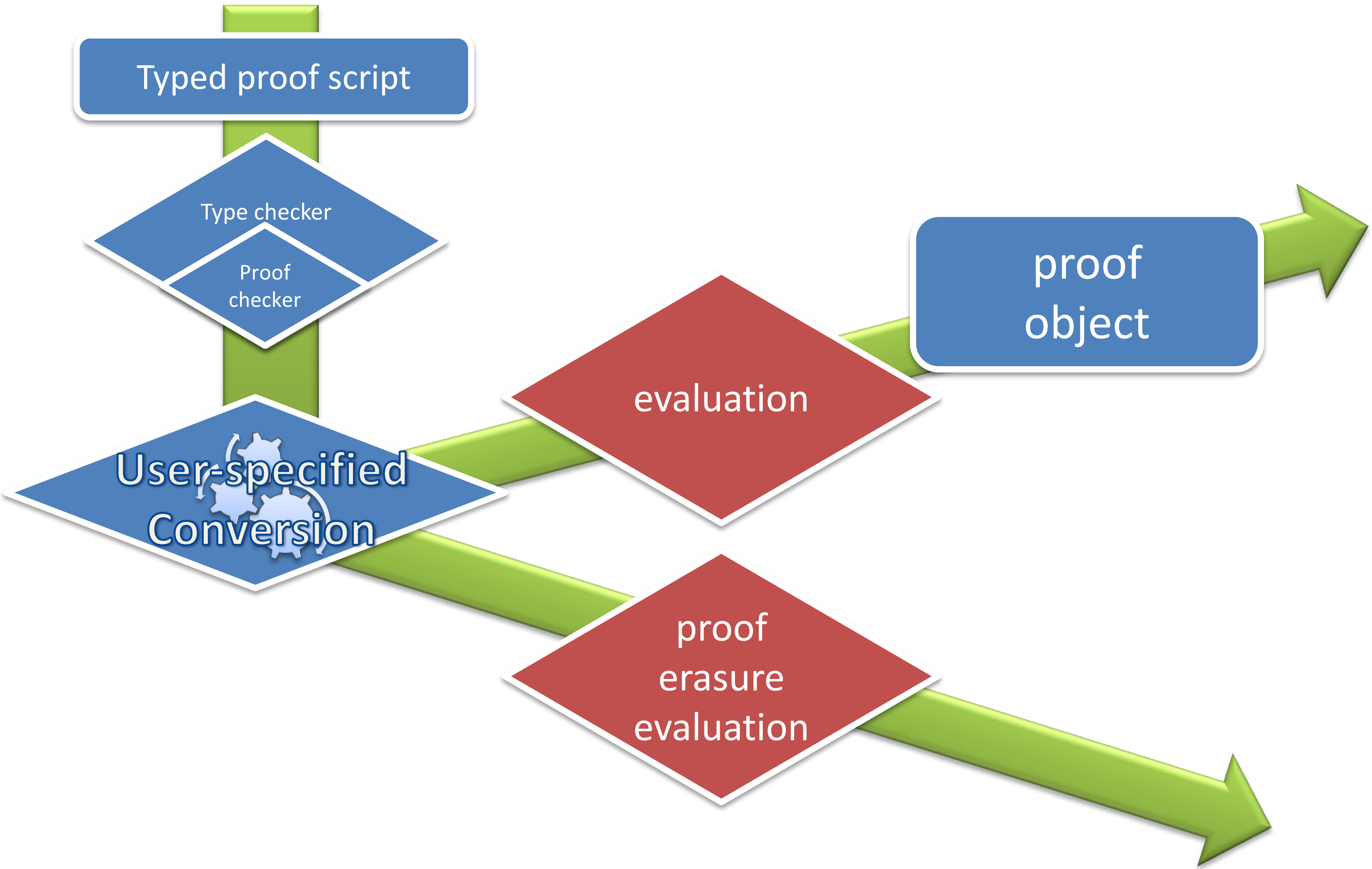

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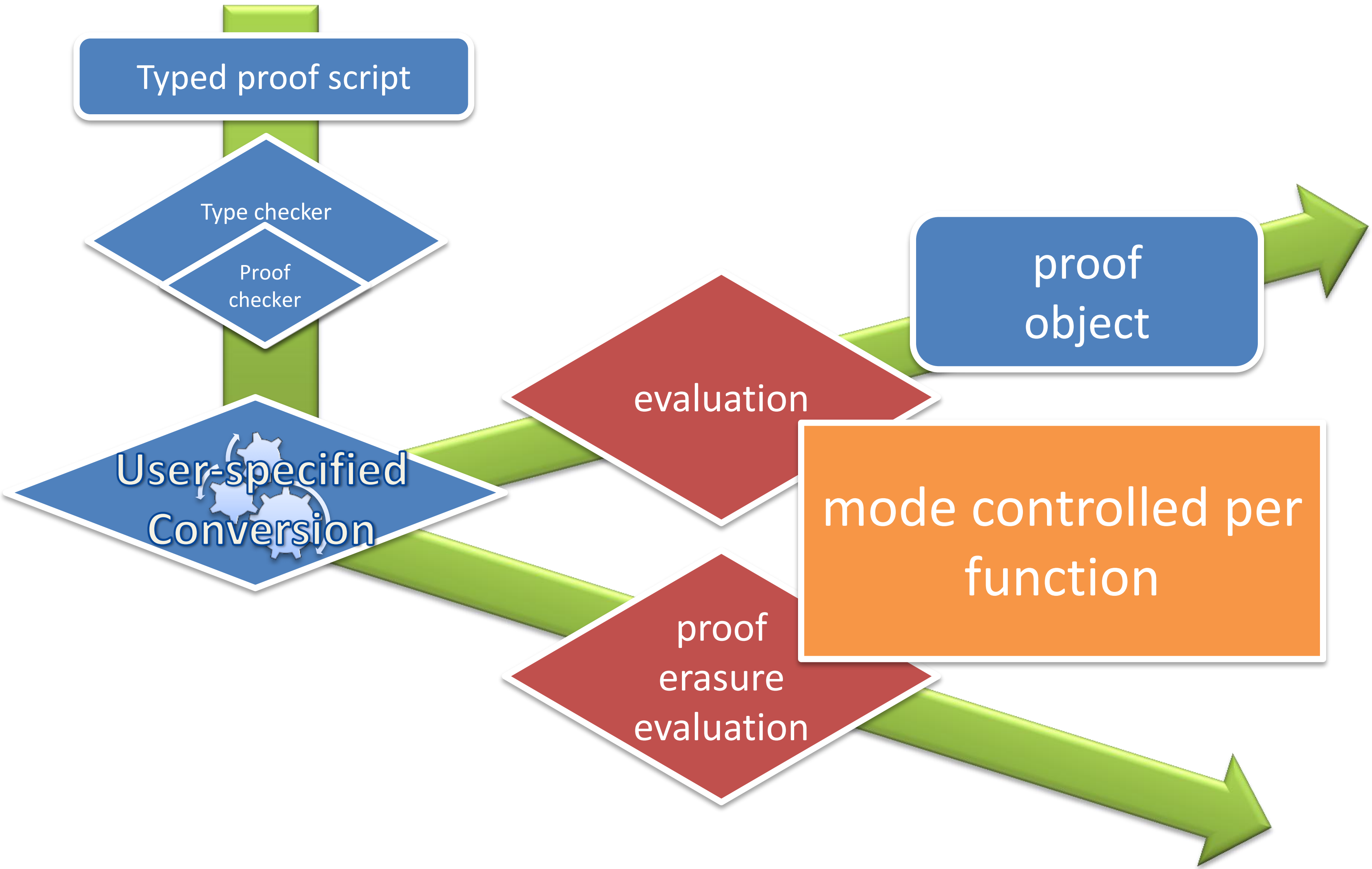
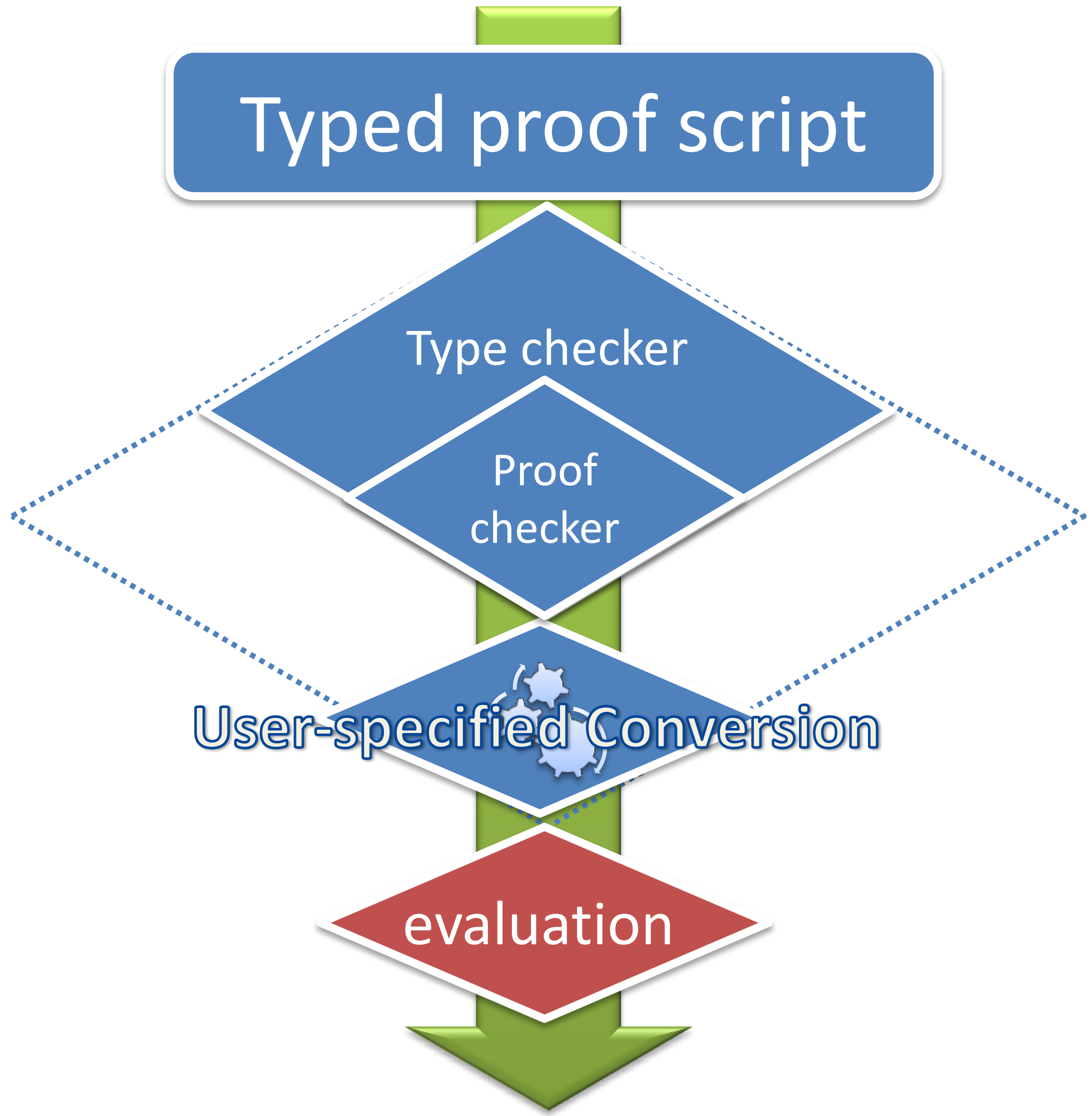


Static checking = type checking + staging under proof-erasure



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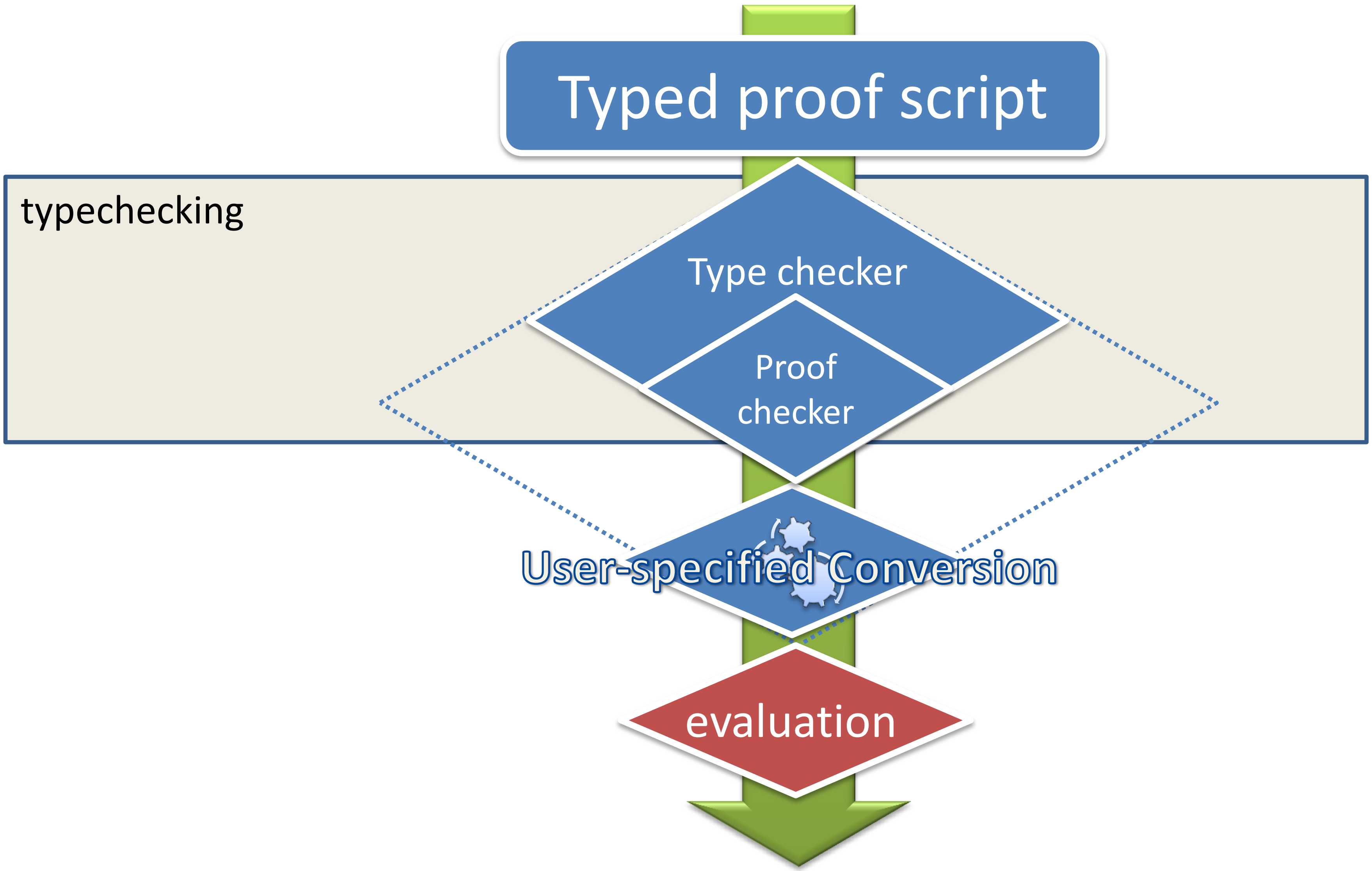

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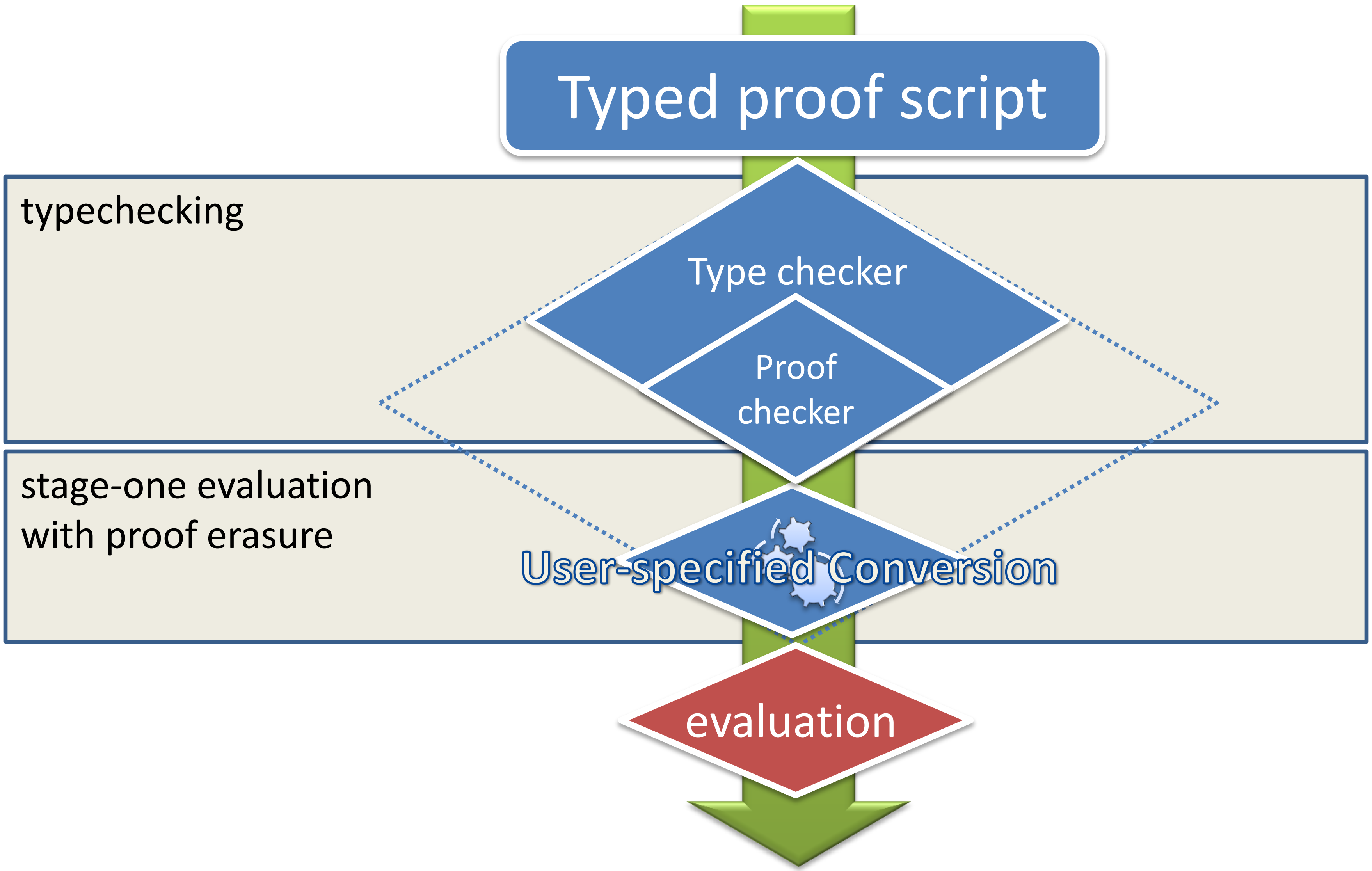

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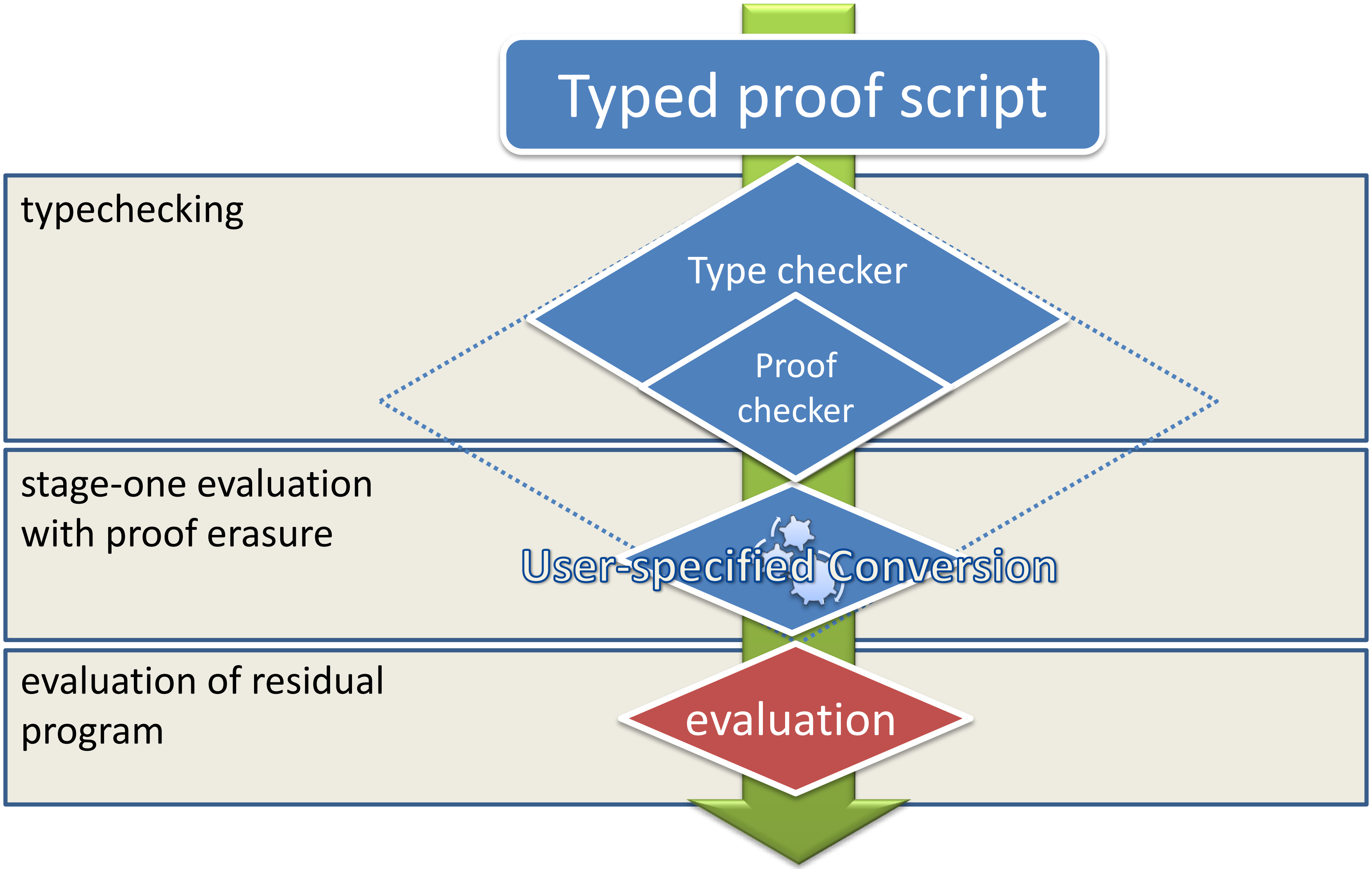
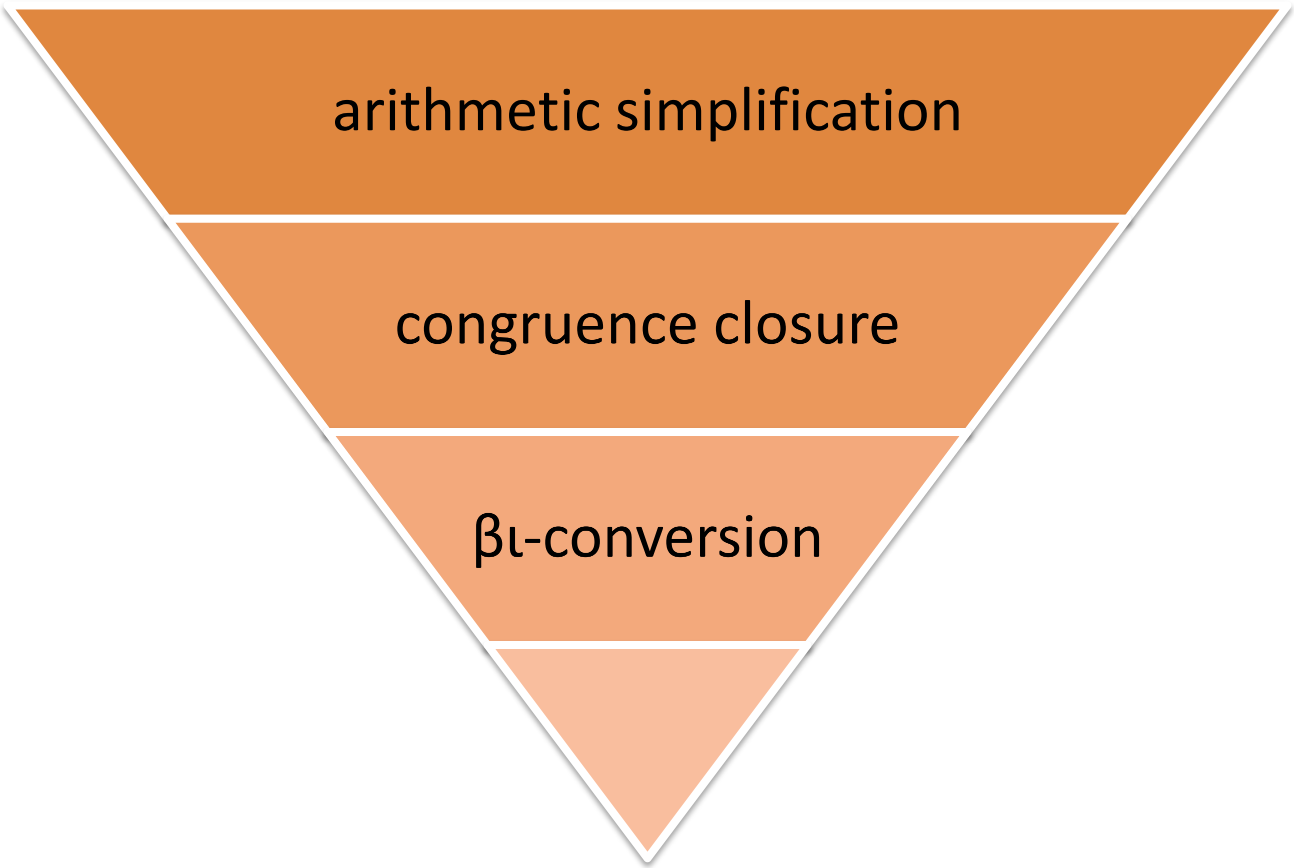


A stack of conversion rules



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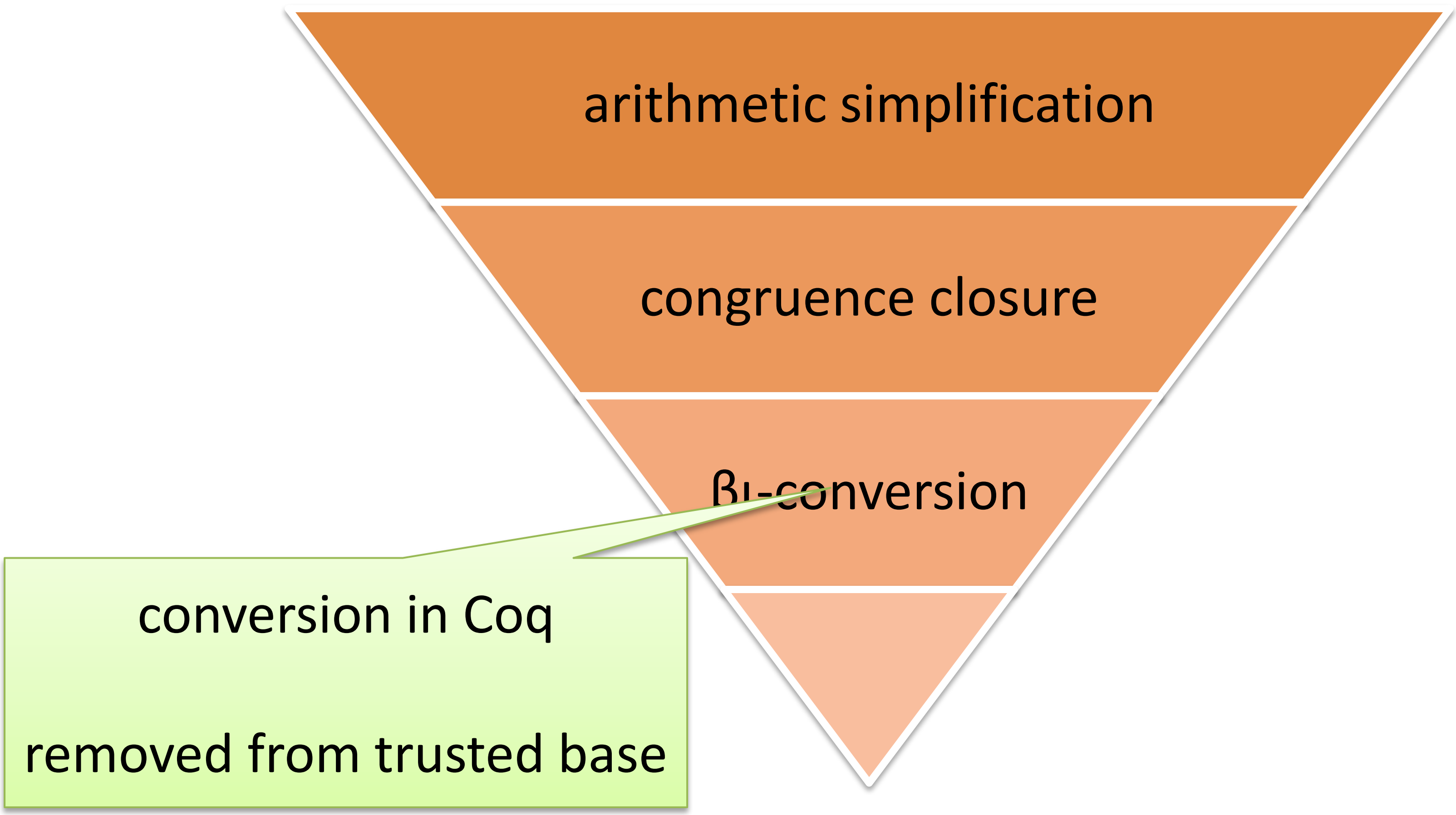

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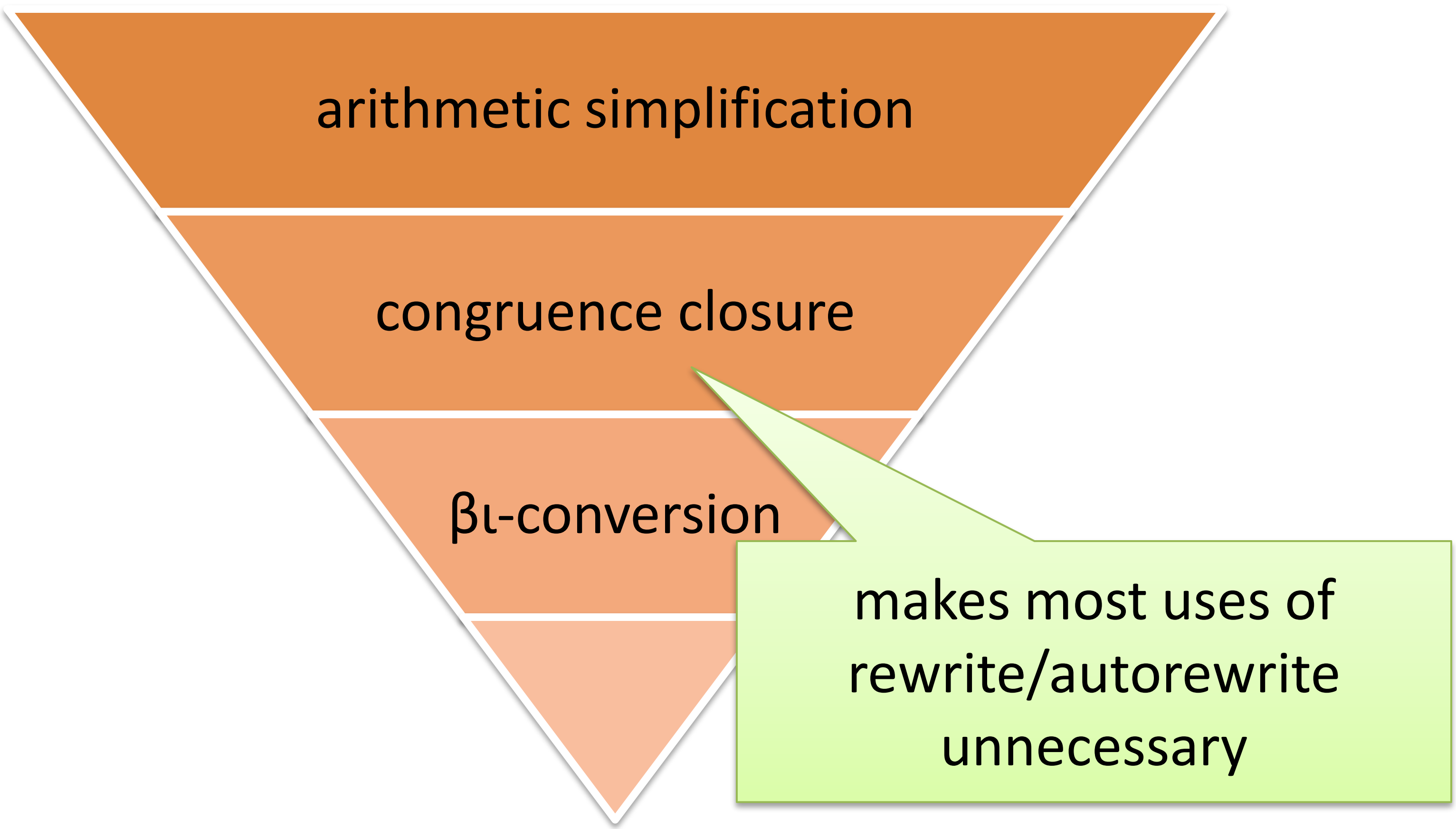

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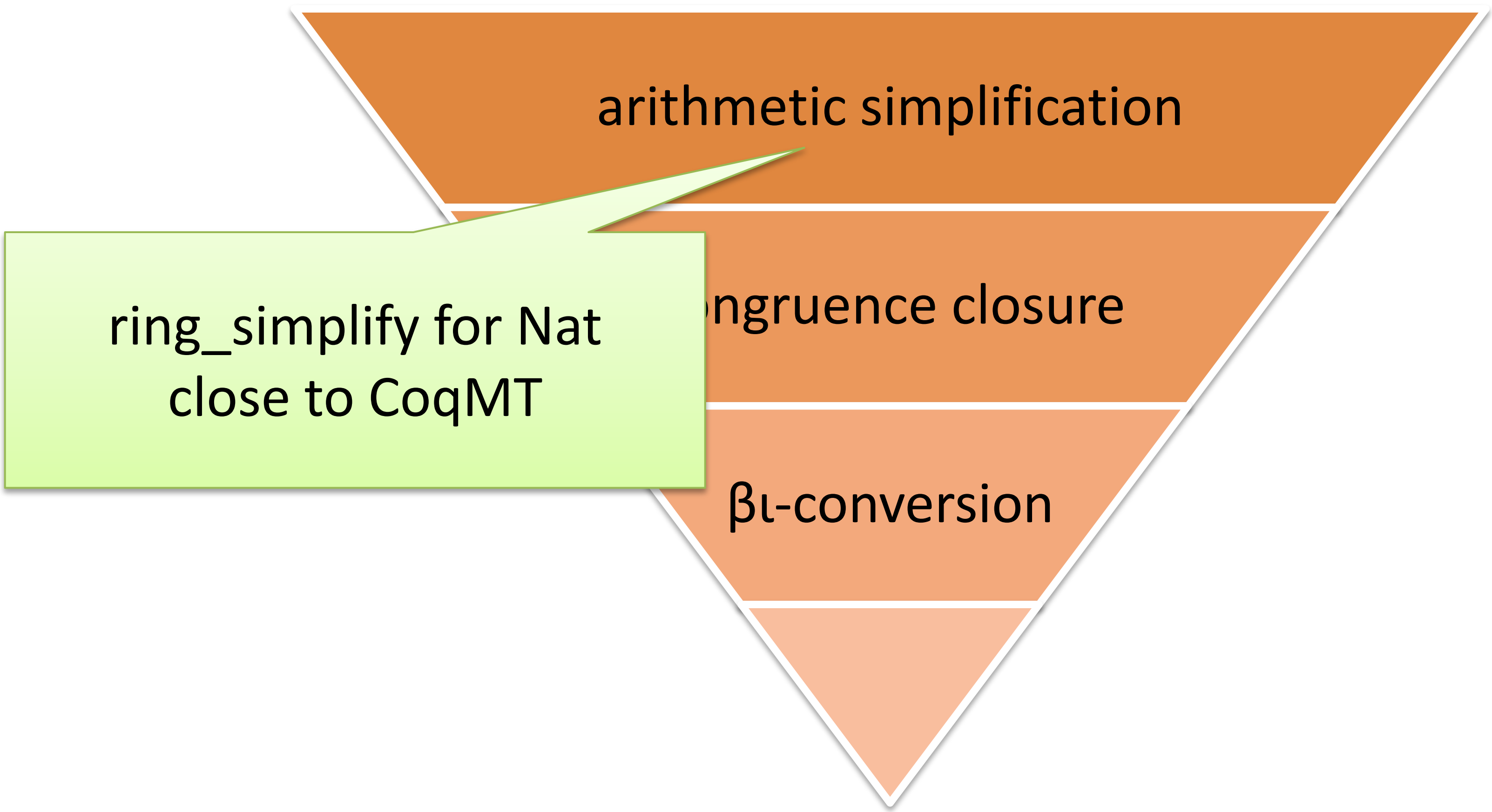

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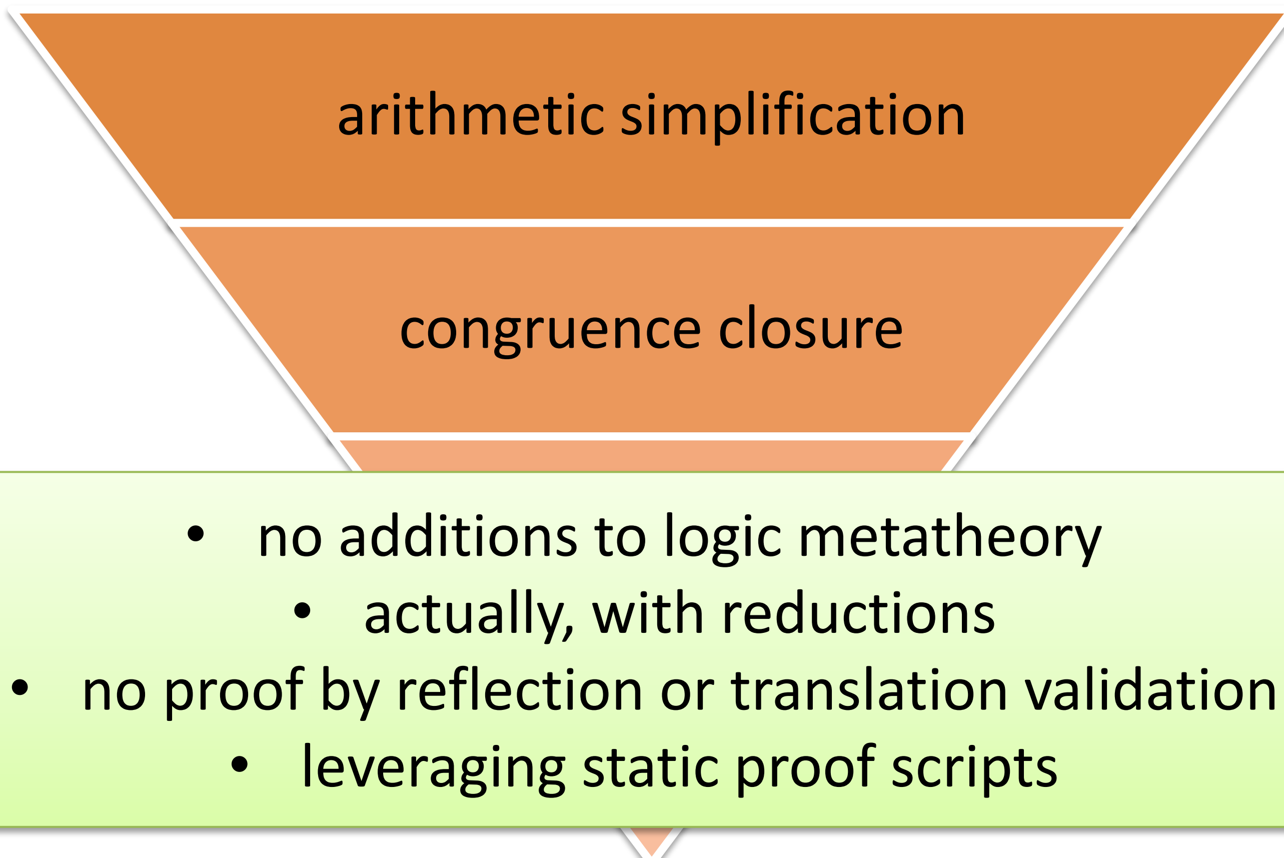
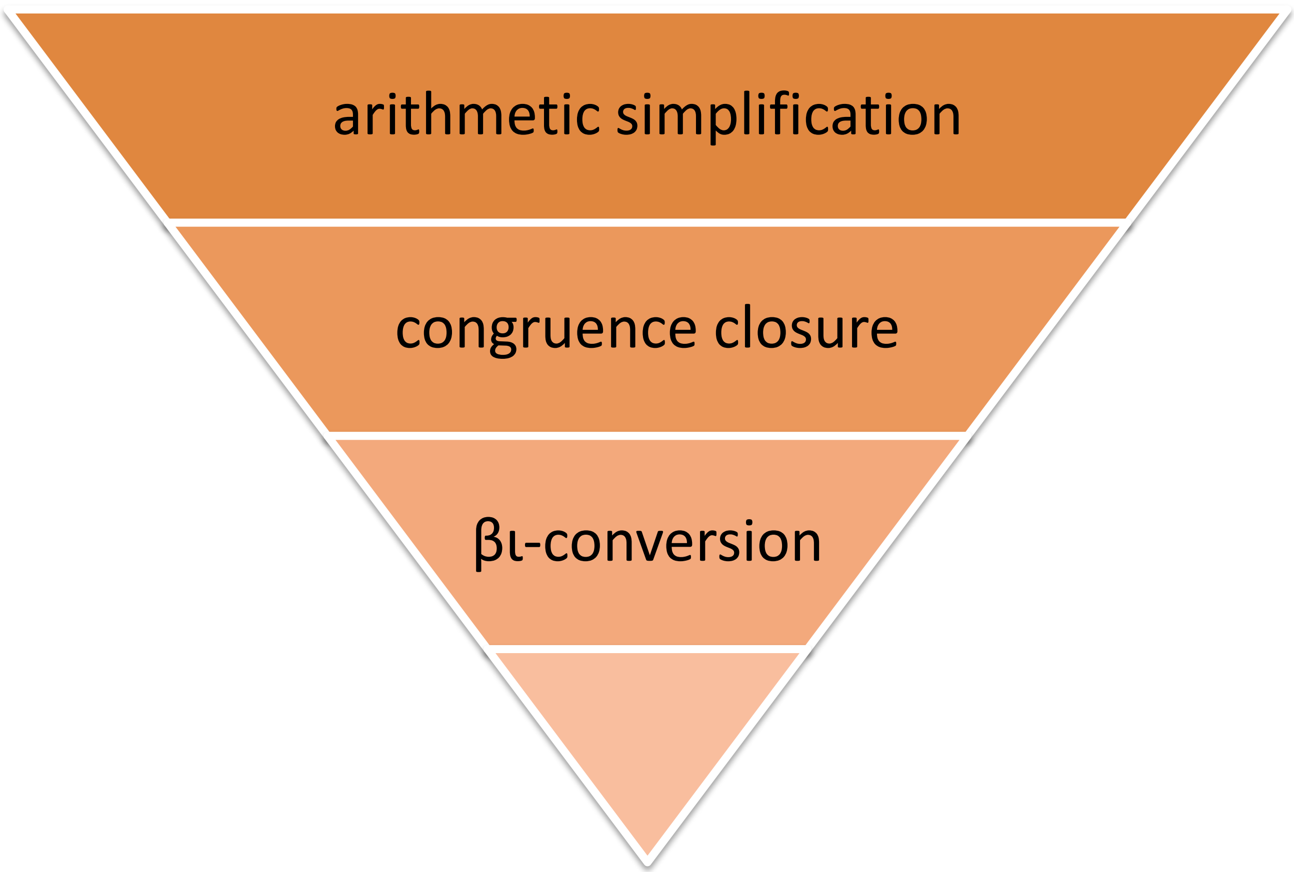


- no additions to logic metatheory
  - actually, with reductions
- no proof by reflection or translation validation
  - leveraging static proof scripts

A stack of conversion rules



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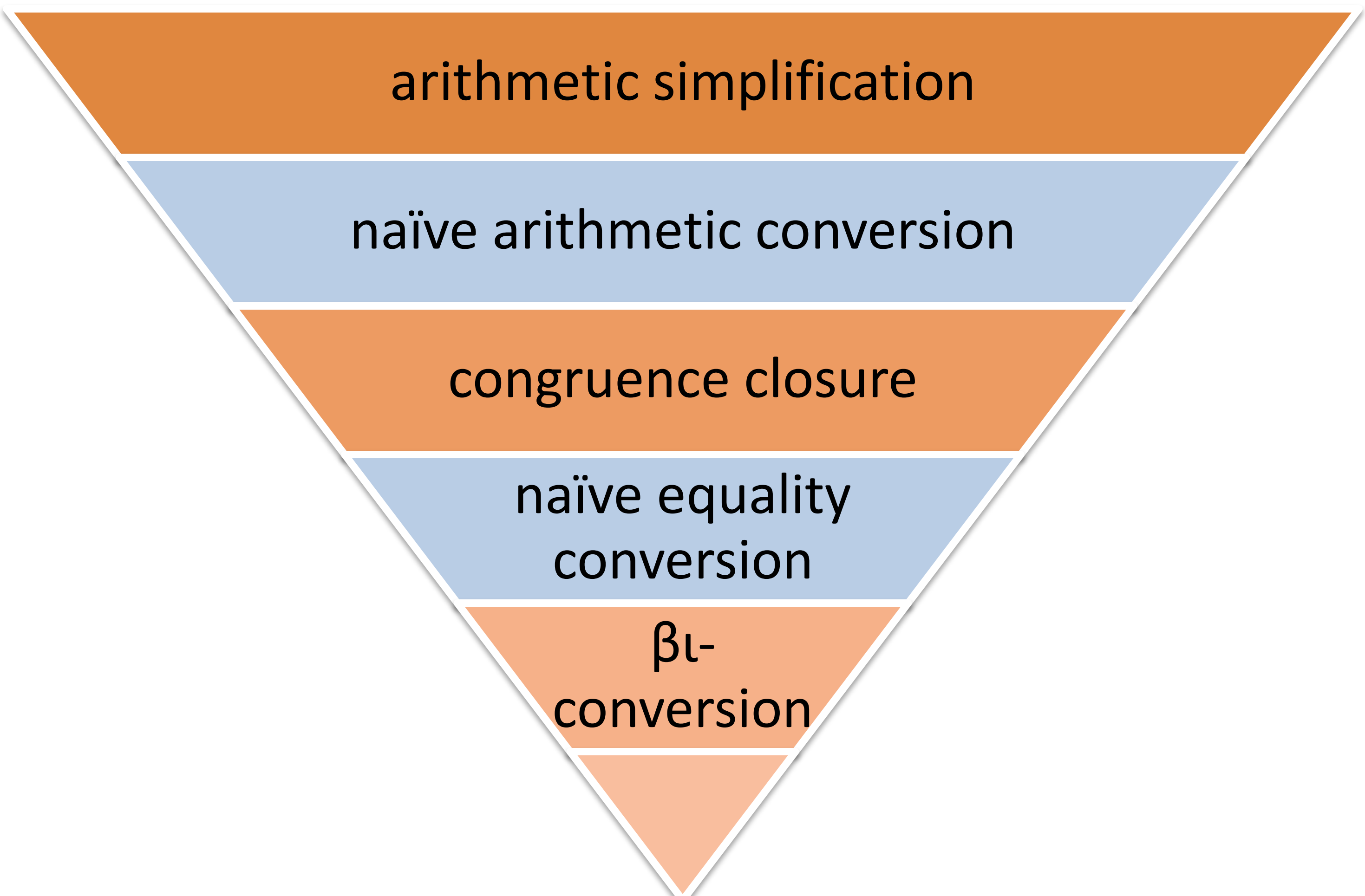

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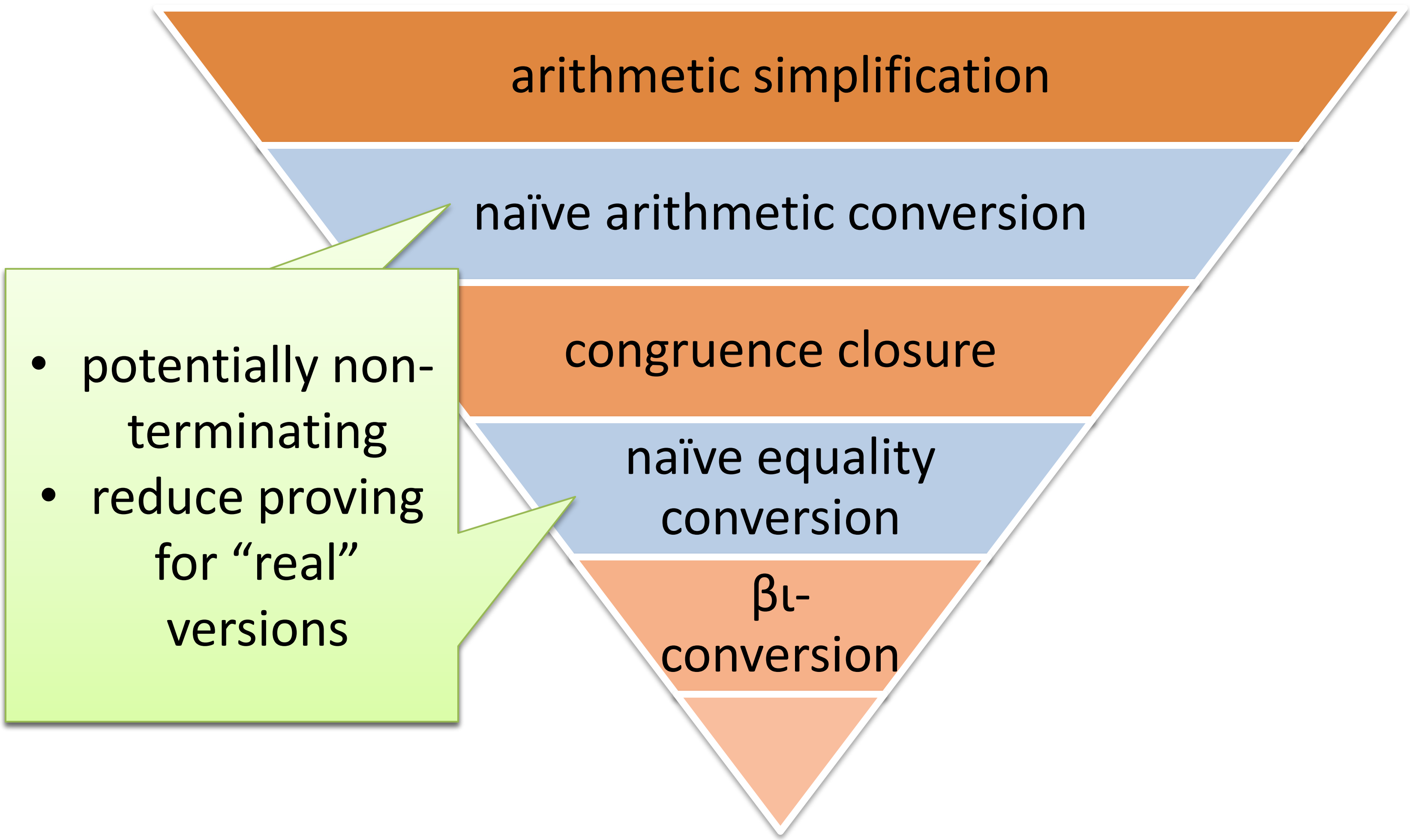

Static proof scripts in tactics

Motivating Coq Example

```
Require Import Arith.
Variable x : Nat.

Theorem test1 : 0 + x = x.
trivial.
Qed.

Theorem test2 : x + 0 = x.
trivial.
Qed.
```

Motivating Coq Example

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Require Import Arith.
Variable x : Nat.

Theorem test1 : 0 + x = x.
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Proof completed

Motivating Coq Example

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Require Import Arith.
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Attempt to save an incomplete proof

Motivating Coq Example

```
Require Import Arith.
Variable x : Nat.

Theorem test1 : 0 + x = x.
trivial.
Qed.

Theorem test2 : x + 0 = x.
trivial.
Qed.
```

Conversion rule can prove this

but can't prove this

Let's add this to our conversion rule!

$$\begin{aligned} \mathsf{lemma1} &: \forall x. x + 0 = x \\ \mathsf{lemma2} &: \forall x y. x + (\mathsf{succ}\ y) = \mathsf{succ}(x + y) \end{aligned}$$

- write a rewriter based on these lemmas
- register it with conversion
- now it's trivial; lemmas used implicitly

$$\begin{aligned} \textsf{lemma1} &: \forall x. x + 0 = x \\ \textsf{lemma2} &: \forall x y. x + (\textsf{succ } y) = \textsf{succ}(x + y) \end{aligned}$$

$$\mathsf{lemma1} \quad : \quad \forall x. x + 0 = x$$

$$\mathsf{lemma2} \quad : \quad \forall x y. x + (\mathsf{succ}\ y) = \mathsf{succ}(x + y)$$

rewriter : $(t : T) \rightarrow (t' : T \times proof : t = t')$

rewriter t =

match t with

$x + y \mapsto$

let y', H = rewriter y in

match y' with

$0 \mapsto x, ?$

$|\ succ\ y'' \mapsto succ\ (x + y''), ?$

$|\ \_ \mapsto t, ?$

$|\ \ldots$

$$\text{lemma1} \quad : \quad \forall x. x + 0 = x$$

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rewriter : $(t : T) \rightarrow (t' : T \times proof : t = t')$

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        $\mid succ\ y'' \mapsto succ\ (x + y''), ?$

        $\mid \_ \mapsto t, ?$

  $\mid \ldots$

$$\frac{\begin{array}{l} x,\ y,\ y'' : T \\ H : y = succ\ y'' \end{array}}{proof : x + y = succ\ (x + y'')}$$

$$\text{lemma1} \quad : \quad \forall x. x + 0 = x$$

$$\text{lemma2} \quad : \quad \forall x y. x + (\text{succ } y) = \text{succ}(x + y)$$

rewriter : $(t : T) \rightarrow (t' : T \times proof : t = t')$

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$$\frac{\begin{array}{l} x,\ y,\ y'' : T \\ H : y = succ\ y'' \end{array}}{proof : x + y = succ\ (x + y'')}$$

lemma2 x y''

$$\text{lemma1} \quad : \quad \forall x. x + 0 = x$$

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rewriter : $(t : T) \rightarrow (t' : T \times proof : t = t')$

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lem… y''

$$\text{lemma1} \quad : \quad \forall x. x + 0 = x$$

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rewriter : $(t : T) \rightarrow (t' : T \times proof : t = t')$

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  match t with

   $x + y \mapsto$

     let $y', \; H \;=\;$ rewriter y

     match y' with

       $0 \qquad\quad \mapsto x, ?$

      $\mid succ\; y'' \mapsto succ\; (x + y''), ?$

      $\mid \_ \qquad\quad \mapsto t, ?$

  $\mid \ldots$

$$\frac{\begin{array}{l} x,\; y,\; y'' : T \\ H : y = succ\; y'' \end{array}}{proof : x + y = succ\; (x + y'')}$$

lemma1 : $\forall x . x + 0 = x$

lemma2 : $\forall x y . x + (\mathsf{succ}\ y) = \mathsf{succ}(x + y)$

lemma2' : $\forall x y y' . y = succ\ y' \rightarrow$
$x + y = succ(x + y')$

$= t')$

$x,\ y,\ y'' : T$
$H : y = succ\ y''$

$proof : x + y = succ\ (x + y'')$

rewriter t =
match t with
$x + y \mapsto$
let y', H = rewriter y
match y' with
$0 \mapsto x, ?$
| $succ\ y'' \mapsto succ\ (x + y''),$ lemma2' $x\ y\ y''$
| $\_ \mapsto t, ?$
| ...

lemma1 : $\forall x.x + 0 = x$

lemma2 : $\forall \; y.x + (\mathsf{succ}\ y) = \mathsf{succ}(x + y)$

lemma2' : $\forall \; y'.y = succ\ y' \rightarrow$

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| $\_ \mapsto t, ?$

| …

$$\text{lemma1} \quad : \quad \forall x. x + 0 = x$$

$$\text{lemma2} \quad : \quad \forall x y. x + (\text{succ } y) = \text{succ}(x + y)$$

$$\text{Exact} \; : \; (proof : P) \to (proof : P')$$

$$\text{rewriter} : (t : T) \to (t' : T \times proof : t = t')$$

$$\frac{\begin{array}{l} x,\; y,\; y'' : T \\ H : y = succ\, y'' \end{array}}{proof : x + y = succ\,(x + y'')}$$

rewriter t =

  match t with

  $x + y \mapsto$

    let y', H = rewriter y

    match y' with

$$\begin{array}{lll} & 0 & \mapsto x, ? \\ | & succ\, y'' & \mapsto succ\,(x + y''), ? \\ | & \_ & \mapsto t, ? \end{array}$$

  | ...

lemma1 : $\forall x. x + 0 = x$

lemma2 : $\forall x y. x + (\text{succ } y) = \text{succ}(x + y)$

Exact : $(proof : P) \to (proof : P')$

rewriter : $(t : T) \to (t' : T \times proof : t = t')$

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      match y' with

        $0 \mapsto x, {?}$

        $\mid succ\ y'' \mapsto succ\ (x + y''), {?}$

        $\mid \_ \mapsto t, {?}$

  $\mid \ldots$

$$\frac{\begin{array}{l} x,\ y,\ y'' : T \\ H : y = succ\ y'' \end{array}}{proof : x + y = succ\ (x + y'')}$$

$$\text{lemma1} \;:\; \forall x. x + 0 = x$$

$$\text{lemma2} \;:\; \forall x y. x + (\text{succ } y) = \text{succ}(x + y)$$

$$\text{Exact} \;:\; (proof : P) \to (proof : P')$$

- similar to trivial
- uses conversion
- P and P' inferred

rewriter $: T) \to (t' : T \times proof : t = t')$

$$\frac{\begin{array}{l} x,\ y,\ y'' : T \\ H : y = succ\ y'' \end{array}}{proof : x + y = succ\ (x + y'')}$$

re

let y', H = rewriter y

match y' with

$$\begin{array}{lll} & 0 & \mapsto x, ? \\ | & succ\ y'' & \mapsto succ\ (x + y''), ? \\ | & \_ & \mapsto t, ? \end{array}$$

$| \ldots$

$\mathsf{lemma1} \quad : \quad \forall x. x + 0 = x$

$\mathsf{lemma2} \quad : \quad \forall x y. x + (\mathsf{succ}\ y) = \mathsf{succ}(x + y)$

$\mathsf{Exact} \ : \ (proof : P) \rightarrow (proof : P')$

$\mathsf{rewriter} : (t : T) \rightarrow (t' : T \times proof : t = t')$

$$\frac{\begin{array}{l} x,\ y,\ y'' : T \\ H : y = succ\ y'' \end{array}}{proof : x + y = succ\ (x + y'')}$$

$\mathsf{rewriter}\ t =$

$\quad \mathsf{match}\ t\ \mathsf{with}$

$\quad\quad x + y \mapsto$

$\quad\quad\quad \mathsf{let}\ y',\ H\ =\ \mathsf{rewriter}\ y$

$\quad\quad\quad \mathsf{match}\ y'\ \mathsf{with}$

$\quad\quad\quad\quad 0 \mapsto x, ?$

$\quad\quad\quad\quad |\ succ\ y'' \mapsto succ\ (x + y''),$

$\quad\quad\quad\quad |\ \_ \mapsto t, ?$

$\quad |\ \ldots$

Exact $(lemma2\ x\ y'')$

$\text{lemma1} \quad : \quad \forall x. x + 0 = x$

$\text{lemma2} \quad : \quad \forall x y. x + (\text{succ } y) = \text{succ}(x + y)$

$\text{Exact} \ : \ (proof : P) \rightarrow (proof : P')$

$\text{rewriter} : (t : T) \rightarrow (t' : T \times proof : t = t')$

$\text{rewriter } t =$

$\quad \text{match } t \text{ with}$

$\quad\quad x + y \mapsto$

$\quad\quad\quad \text{let } y', \ H \ = \ \text{rewriter } y \text{ i}$

$x, \ y, \ y'' : T$

$H : y = succ\ y''$

$$\frac{\begin{array}{l} x, \ y, \ y'' : T \\ H : y = succ\ y'' \end{array}}{proof : x + y = succ\ (x + y'')}$$

- not checked statically
- recomputed many times

$\quad\quad\quad | \ succ\ y'' \mapsto succ\ (x + y''),$ $\text{Exact } (lemma2\ x\ y'')$

$\quad\quad\quad | \ \_ \quad\quad \mapsto t, ?$

$\quad | \ \ldots$

$$\text{lemma1} \quad : \quad \forall x. x + 0 = x$$

$$\text{lemma2} \quad : \quad \forall x y. x + (\text{succ } y) = \text{succ}(x + y)$$

$$\text{Exact} \; : \; (proof : P) \rightarrow (proof : P')$$

$$\text{rewriter} : (t : T) \rightarrow (t' : T \times proof : t = t')$$

rewriter t =

  match t with

    $x + y \mapsto$

      let $y', \; H \; = \;$ rewriter y

      match y' with

        $0 \qquad\qquad \mapsto x, ?$

        $\mid succ\ y'' \mapsto succ\ (x + y''), ?$

        $\mid \_ \qquad\qquad \mapsto t, ?$

  $\mid \ldots$

$$\frac{x,\ y,\ y'' : T \qquad H : y = succ\ y''}{proof : x + y = succ\ (x + y'')}$$

$$\mathsf{lemma1} \quad : \quad \forall x. x + 0 = x$$

$$\mathsf{lemma2} \quad : \quad \forall x y. x + (\mathsf{succ}\ y) = \mathsf{succ}(x + y)$$

$$\mathsf{Exact}\ :\ (proof : P) \to (proof : P')$$

$$\mathsf{rewriter}\ :\ (t : T) \to (t' : T \times proof : t = t')$$

$$
\begin{array}{l}
\mathsf{rewriter}\ t = \\
\quad \mathsf{match}\ t\ \mathsf{with} \\
\quad\quad x + y \mapsto \\
\quad\quad\quad \mathsf{let}\ y',\ H \ = \ \mathsf{rewriter}\ y\ \mathsf{i} \\
\quad\quad\quad \mathsf{match}\ y'\ \mathsf{with} \\
\quad\quad\quad\quad 0 \qquad\quad \mapsto x, {\color{red}?} \\
\quad\quad\quad\ \mid succ\ y'' \mapsto succ\ (x + y''), \\
\quad\quad\quad\ \mid \_ \qquad\quad \mapsto t, {\color{red}?} \\
\quad \mid \ldots
\end{array}
$$

$$
\begin{array}{l}
x,\ y,\ y'' : T \\
H : y = succ\ y'' \\
\hline\hline
proof : x + y = succ\ (x + y'')
\end{array}
$$

$$
\left\{ \begin{array}{l} \mathsf{Exact} \\ (lemma2\ x\ y'') \end{array} \right\}_{static}
$$

$$\text{lemma1} \quad : \quad \forall x. x + 0 = x$$

$$\text{lemma2} \quad : \quad \forall x y. x + (\text{succ } y) = \text{succ}(x + y)$$

$$\text{Exact} \ : \ (proof : P) \to (proof : P')$$

$$\text{rewriter} : (t : T) \to (t' : T \times proof : t = t')$$

rewriter t =

match t with

$x + y \mapsto$

$$\begin{array}{l} x,\ y,\ y'' : T \\ H : y = succ\ y'' \\ \hline\hline proof : x + y = succ\ (x + y'') \end{array}$$

- checked at definition time
- computed once
- transformation of runtime arguments to constant arguments

$+ y''),$

$$\left\{\begin{array}{l}\text{Exact} \\ (lemma2\ x\ y'')\end{array}\right\}_{static}$$

$| \ldots$

How does it work?

rewriter : $\begin{array}{l}(\Phi : \mathsf{ctx}) \longrightarrow (T : [\Phi]\,\mathsf{Type}) \longrightarrow (t : [\Phi]\,T) \\ \longrightarrow (t' : [\Phi]\,T \times proof : [\Phi]\,t = t')\end{array}$

rewriter Φ t $=$

match t with

$[\Phi]\,x + y \mapsto$

let $[\Phi]\,y',\ [\Phi]\,H\ =$ rewriter $[\Phi]\,y$ in

match y' with

$$\begin{array}{lll} & [\Phi]\,0 & \mapsto [\Phi]\,x, ? \\ | & [\Phi]\,succ\ y'' & \mapsto [\Phi]\,succ\ (x + y''), \left\{\begin{array}{l}\text{Exact} \\ (lemma2\ x\ y'')\end{array}\right\}_{static} \\ | & \_ & \mapsto [\Phi]\,t, ? \end{array}$$

$| \ \ldots$

How does it work?

$$\text{Exact} \;:\; (\Phi : \mathsf{ctx}) \to (proof : [\Phi]\, P) \to (proof : [\Phi]\, P')$$

$$\text{rewriter} : \boxed{\begin{array}{l}(\Phi : \mathsf{ctx}) \to (T : [\Phi]\, \mathsf{Type}) \to (t : [\Phi]\, T) \\ \to (t' : [\Phi]\, T \times proof : [\Phi]\, t = t')\end{array}}$$

$$\begin{array}{l}
\text{rewriter } \Phi\ t = \\
\quad \text{match } t \text{ with} \\
\quad\quad [\Phi]\, x + y \mapsto \\
\quad\quad\quad\quad \text{let } [\Phi]\, y',\ [\Phi]\, H \ = \ \text{rewriter } [\Phi]\, y \text{ in} \\
\quad\quad\quad\quad \text{match } y' \text{ with} \\
\quad\quad\quad\quad\quad \begin{array}{lll} & [\Phi]\, 0 & \mapsto [\Phi]\, x, {\color{red}?} \\ | & [\Phi]\, succ\ y'' & \mapsto [\Phi]\, succ\ (x + y''), \left\{ \begin{array}{l} \text{Exact} \\ (lemma2\ x\ y'') \end{array} \right\}_{static} \\ | & _ & \mapsto [\Phi]\, t, {\color{red}?} \end{array} \\
\quad | \ \ldots
\end{array}$$

$$
\begin{array}{l}
\textsf{letstatic}\ pf\ =\\
\qquad \textsf{let}\ \Phi'\ =\ [x, y, y'' : \textsf{Nat}, H : y = succ\ y'']\ \textsf{in}\\
\qquad \textsf{Exact}\ \Phi'\ ([\Phi']\, lemma2\ x\ y'')\\
\textsf{in}\\
[\Phi]\, pf\,/[x/\textsf{id}_\Phi, y/\textsf{id}_\Phi, y'/\textsf{id}_\Phi, H/\textsf{id}_\Phi]
\end{array}
$$

Exact ...)

rewriter : ...

$$
\begin{array}{l}
\textsf{rewriter}\ \Phi\ t =\\
\quad \textsf{match}\ t\ \textsf{with}\\
\quad\quad [\Phi]\, x + y \mapsto\\
\quad\quad\quad \textsf{let}\ [\Phi]\, y',\ [\Phi]\, H\ =\ \textsf{rewriter}\ [\Phi]\, y\ \textsf{in}\\
\quad\quad\quad \textsf{match}\ y'\ \textsf{with}\\
\quad\quad\quad\quad [\Phi]\, 0 \mapsto [\Phi]\, x, ?\\
\quad\quad\quad\ \ \mid [\Phi]\, succ\ y'' \mapsto [\Phi]\, succ\ (x + y''), \left\{\begin{array}{l}\textsf{Exact}\\ (lemma2\ x\ y'')\end{array}\right\}_{static}\\
\quad\quad\quad\ \ \mid \_ \mapsto [\Phi]\, t, ?\\
\quad \mid \ldots
\end{array}
$$

Exact

rewriter :

rewriter Φ t =

match t w

$[\Phi]\; x +$

let $H = $ rewriter $[\Phi]\; y$ in

$2\; x\; y'')\}_{static}$

$|\; \_ \quad \mapsto [\Phi]\; t, ?$

$|\; \ldots$

letstatic pf =

let $\Phi' = [x, y, y'' : \mathsf{Nat}, H : y = succ\; y'']$ in

Exact Φ' $([\Phi']\; lemma2\; x\; y'')$

in

$[\Phi \ldots][x/\mathsf{id}_\Phi, y/\mathsf{id}_\Phi, y'/\mathsf{id}_\Phi, H/\mathsf{id}_\Phi]$

$$\frac{\bullet; \Sigma; \Gamma|_{\text{static}} \vdash e : \tau \qquad \Psi; \Sigma; \Gamma, x :_s \tau \vdash e' : \tau}{\Psi; \Sigma; \Gamma \vdash \text{letstatic}\; x = e \text{ in } e' : \tau}$$

Implementation

http://www.cs.yale.edu/homes/stampoulis/

- type inferencing and implicit arguments
- compiler to OCaml
- rewriter code generation
- inductive types

Talk to me for a demo!

What's in the paper and TR

- Static and dynamic semantics
- Metatheory:
  Type-safety theorem
  Proof erasure theorem
  Static proof script transformation
- Implementation details and examples implemented
- Typed proof scripts as flexible proof certificates

Related work

- proof-by-reflection
  - *restricted programming model (total functions)*
  - *tedious to set up*
  - *here: no need for termination proofs*
- automation through canonical structures / unification hints
  - *restricted programming model (logic programming)*
  - *very hard to debug*

Summary

- a new architecture for proof assistants
- user-extensible checking of proofs and tactics
- minimal trusted core
- reduce required effort for formal proofs

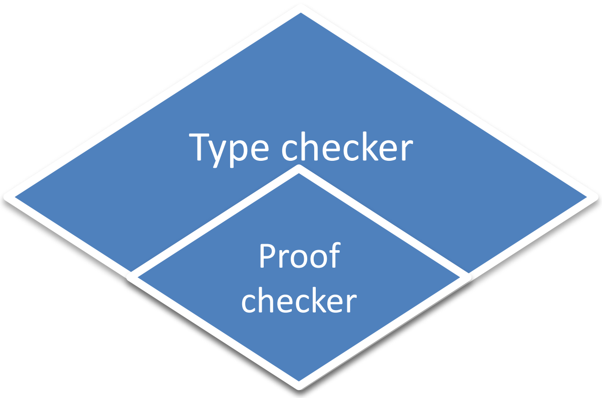
Thanks!

http://www.cs.yale.edu/homes/stampoulis/

Backup slides

Type checking proofs and tactics

-manipulate proofs and propositions in a type-safe manner
-dependent pattern matching on logical terms
-logic and computation are kept separate
-Beluga *[Pientka & Dunfield ‘08]*
-Delphin *[Poswolsky & Schürmann ‘08]*
-VeriML *[Stampoulis & Shao ‘10]*



Related work

- LCF family of proof assistants
  - *no information while writing proof scripts/tactics*
- Coq / CoqMT
  - *conversion rule is fixed*
  - *changing it requires re-engineering*
- NuPRL
  - *extensional type theory and sophisticated conversion*
  - *here: user decides conversion (level of undecidability)*
- Beluga / Delphin
  - *use as metalogic for LF*
  - *here: the logic is fixed; the language is the proof assistant*